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A test, using atmospheric data, of a method for estimating oceanic eddy diffusivity

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Abstract. G. Holloway has proposed a simple method to estimate the effective horizontal eddy diffusivity for oceanic tracers using satellite measurements of sea-surface height variability. In this method, the diffusivity is assumed to scale as the r.m.s. eddy geostrophic streamfunction. This method should apply analogously in an atmospheric context, in which the energetic eddies are well resolved in data sets and models, and in which both the effective diffusivity and tracer fluxes can be directly calculated. We evaluate Holloway's method using lower-tropospheric daily atmospheric reanalysis data, and find that at a given height the method succesfully yields estimates of the divergent part of the atmospheric sensible heat flux, to within a fairly uniform scaling factor.

Introduction

An ongoing problem in ocean climate modelling is the need to parameterize tracer transport by mesoscale eddies. Although recent global ocean models attempt to resolve the mesoscale eddies [*Semtner and Chervin*, 1992], such models remain impractical for most climate research. Furthermore, the mesoscale-eddy statistics and transports are generally not well characterized by in situ field measurements. With the arrival of global high-resolution sea-surface height variability data based on satellite altimetry, however, our knowledge of the mesoscale eddies at the sea surface has improved substantially (e.g. *Stammer* [1997] and references therein).

Holloway [1986] (see also *Keffer and Holloway* [1988] and *Stammer* [1998]) has proposed that altimeter-based maps of the sea-surface height variability provide effective maps of the horizontal eddy diffusivity. The diffusivity can then be used to estimate eddy transport. The idea can be put most simply in terms of the following scaling argument. The diffusivity scales as $\mathcal{D} \sim v\ell$, where v is r.m.s. eddy velocity and l is a mixing length. The geostrophic relation yields variations of the geostrophic streamfunction, ψ, from variations of the surface height, h:

$$gh/f = \psi, \tag{1}$$

where g is the gravitational acceleration and f the Coriolis parameter. Since the velocity field is given by $\vec{u} = (\partial_x\psi, -\partial_y\psi)$, the r.m.s. eddy streamfunction can be expressed as an r.m.s. eddy velocity times a length scale. Setting this length scale proportional to the mixing length, ℓ, we have

$$\mathcal{D} = \alpha\left[\overline{(\psi'^2)}\right]^{1/2} = \alpha s(\psi), \tag{2}$$

where α is the coefficient of proportionality; the overbar denotes a time mean; the prime denotes an eddy quantity, that is, a deviation from the time mean; and $s(A)$ is the standard deviation of the quantity A. The diffusivity and r.m.s. eddy streamfunction have the same dimensions.

Although this diffusivity estimate is not a parameterization of eddy transport properties, it is a potentially useful way of obtaining these properties from available data. It therefore seems worthwhile to test the estimate directly. This is difficult to do in an oceanic framework without high-resolution tracer flux data. On the other hand, such data can be obtained from standard atmospheric data sets and models, in which the energy-containing eddies are very well resolved.

We here describe a test of Holloway's method that uses atmospheric data to calculate both the atmospheric eddy diffusivity and a representative tracer flux, namely the flux of sensible heat. Our working assumption is that this method should apply equally well to atmospheric and oceanic geostrophic turbulence. However, in contrast to Holloway, who uses the diffusivity estimate (2) to obtain vertically integrated tracer fluxes, we emphasize that (2) should be understood as a measure of the *near-surface* horizontal diffusivity. We return to this last point in the discussion.

Data-analysis method

We use the NCEP/NCAR four-times-daily 2.5°×2.5° global reanalysis data set for the years 1979–1995 [*Kalnay et al.*, 1996]. We analyze data at 850 mb, which is characteristic of the free lower troposphere, and at 1000 mb, which is characteristic of the near-surface boundary layer. Vector fields are decomposed into their rotational and divergent parts using a spectral transforms package that requires the data to be interpolated onto an equivalent-resolution Gaussian grid. The eddy statistics discussed below are computed using a time series that is adjusted by removing the climatological seasonal cycle. The climatological seasonal cycle is obtained by averaging data at a given month, day and six-hour period in a calendar year over the 17 years of the data set, and by then passing the resulting year-long time series through a 31-day running-average smoother. The adjusted time series retains interannual-timescale variability. Eddy quantities are computed with respect to either the 17-year mean or the 17-year seasonal mean of the adjusted time series. The mean temperature fields and temperature gradients are calculated from the original unadjusted time series.

Diffusivity estimate

We assume that the flow is predominantly horizontal, incompressible and geostrophic with streamfunction ψ given

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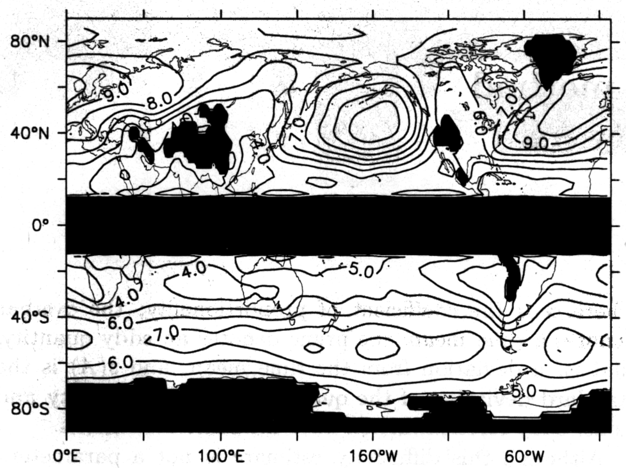

Figure 1. DJF-averaged 850 mb $s(\psi)$. Contour interval: 10^6 m^2/s. In these figures, regions where the mean surface pressure is considerably less than 850 mb are masked. (Specifically, masking is applied where the mean height of the pressure surface is 200 m or more below the earth's surface.) Regions equatorward of 10°N and 10°S are also masked, since the geostrophic streamfunction (1) diverges at the equator.

by (1). The climatological DJF-mean r.m.s. eddy streamfunction, $s(\psi)$, is shown in Figure 1. Thinking of the map of the r.m.s. eddy streamfunction as a map of the horizontal diffusivity, we see that the diffusivity is spatially inhomogenous, with maxima in the Northern (winter) hemisphere over the eastern ocean basins. The areal average r.m.s. eddy streamfunction poleward of 30°N and 30°S is 6.7×10^6 m^2/s. By comparison, the value of thermal diffusivity used by simple energy-balance climate models that include both atmospheric and oceanic heat transport is typically 3–5 $\times 10^6$ m^2/s (e.g. *North* [1975]). We therefore expect that in (2) the scaling factor α is less than unity but order unity.

If the heat transport is horizontal, downgradient and isotropic, the heat flux satisfies

$$\overline{\vec{u}'T'} = -\mathcal{D}\vec{\nabla}\overline{T} = -\alpha s(\psi)\vec{\nabla}\overline{T}, \qquad (3)$$

where $\vec{u}$ is the horizontal wind, T is the temperature, and where we have used (2). We now test the validity of (3) by comparing $\overline{\vec{u}'T'}$ to $-\alpha s(\psi)\vec{\nabla}\overline{T}$. Since only the divergence of the tracer flux is important for net tracer transport, we examine only the divergent parts of $\overline{\vec{u}'T'}$ and $-s(\psi)\vec{\nabla}\overline{T}$. The rotational component of horizontal tracer fluxes has been discussed elsewhere [*Lau and Wallace*, 1979; *Marshall and Shutts*, 1981]; it is at least partially related to the mean-flow advection of tracer variance, and cannot generally be expected to be directed down gradient.

The divergent part of the 850 mb DJF sensible heat flux, $\overline{\vec{u}'T'}$, where the overbar indicates the DJF climatological average, is shown by the red vectors in Figure 2, overlain on contours of the DJF climatological-mean temperature. Also shown in the figure, with blue vectors that are scaled a factor three smaller than the red vectors, is the divergent part of $-s(\psi)\vec{\nabla}\overline{T}$. Both fields are predominantly downgradient and therefore well aligned. Referring to the relative scaling of the vectors in Figure 2, the figure suggests a value for the scale factor α in the range 0.3–0.5.

To estimate α quantitatively, we perform a linear regression between the divergent parts of $\overline{\vec{u}'T'}$ and $-s(\psi)\vec{\nabla}\overline{T}$. The results are shown in Table 1 (see caption for details). We carry out a linear regression for the meridional components, for each of the climatological seasonal and annual means. As a cross check, we perform a separate regression for the zonal components. We exclude the polar and tropical regions where the estimate breaks down, as is evident in Figure 2.

For the DJF data, we obtain $\alpha \approx 0.34$ for the meridional divergent component of (3), with a correlation coefficient of $r^2 \approx 0.93$. (For the extratropical-average $s(\psi)$ value of 6.7×10^6 m^2/s noted above, this value of α results in an average diffusivity of 2.3×10^6 m^2/s.) We obtain $\alpha \approx 0.42$ for the zonal divergent component, but the fit is poorer, with $r^2 \approx 0.63$. Table 1 shows that the estimates of α are fairly robust, varying little over the wide variety of conditions represented by the seasonal evolution of the mean baroclinicity in each hemisphere. The zonal component of the flux is relatively noisy and is probably not well modelled by (3). Nevertheless, since the zonal component tends to be smaller than the meridional component, we conclude that (3) is a fairly good model of near-surface horizontal heat transport in the atmosphere.

To obtain a clearer sense of the quality of the estimate, we show in Figure 3a the magnitude of the divergent part of the meridional component of the heat flux, and in Figure 3b the estimate of this field using the value of $\alpha_{DJF} = 0.34$ from Table 1. The estimate reproduces the main features of the heat-flux pattern quite well over the Pacific, Atlantic and Southern-Hemisphere midlatitudes. The estimate also reproduces the relative strengths of the maxima fairly well. Overall, the results are encouraging.

A more direct verification that $s(\psi)$ in (3) and in Figure 1 represents a map of the diffusivity is shown in Figure 4. This figure is a map, for the DJF data, of the ratio of the magnitude of the divergent part of the heat flux to $\alpha_{DJF}|\vec{\nabla}\overline{T}|$ (where $\alpha_{DJF} = 0.34$), that has been smoothed as described in the caption. This quantity, apart from the scaling by α_{DJF}, is a rough measure of the local diffusivity;

Table 1. Scale factor α in (3) and correlation coefficient r^2 for linear regression of model equation $y = \alpha^{-1}x$+constant. Here, independent variable x represents a component of the divergent part of the heat flux and dependent variable y the corresponding component of $-s(\psi)\vec{\nabla}\overline{T}$. Results are shown for 850 mb and 1000 mb, for zonal (ZON) and meridional (MER) components,and for seasonal (DJF, MAM, JJA, SON) and annual (ANN) data. Calculations use above-ground points in the 30–70° latitude bands and weighting to account for the spherical geometry. Over 4,400 points were used for the 850 mb calculations and over 3,400 points for the 1000 mb calculations.

	850 mb				1000 mb			
	ZON		MER		ZON		MER	
	α	r^2	α	r^2	α	r^2	α	r^2
DJF	0.42	0.63	0.34	0.93	0.17	0.65	0.18	0.89
MAM	0.40	0.59	0.36	0.89	0.17	0.57	0.20	0.89
JJA	0.43	0.49	0.35	0.85	0.18	0.54	0.18	0.87
SON	0.44	0.50	0.38	0.93	0.19	0.45	0.20	0.92
ANN	0.42	0.58	0.36	0.93	0.18	0.58	0.20	0.93

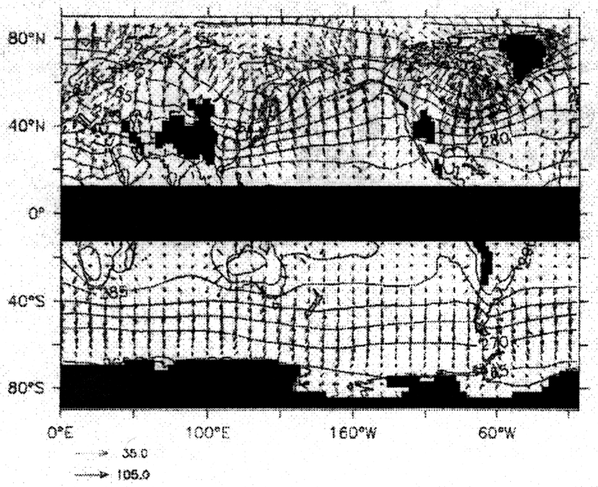

Figure 2. Red vectors: DJF-averaged divergent part of the 850 mb heat flux. Units: mK/s. Blue vectors: the divergent part of $-s(\psi)\vec{\nabla}T$. Contours: DJF-averaged mean temperature; contour interval: 5 K.

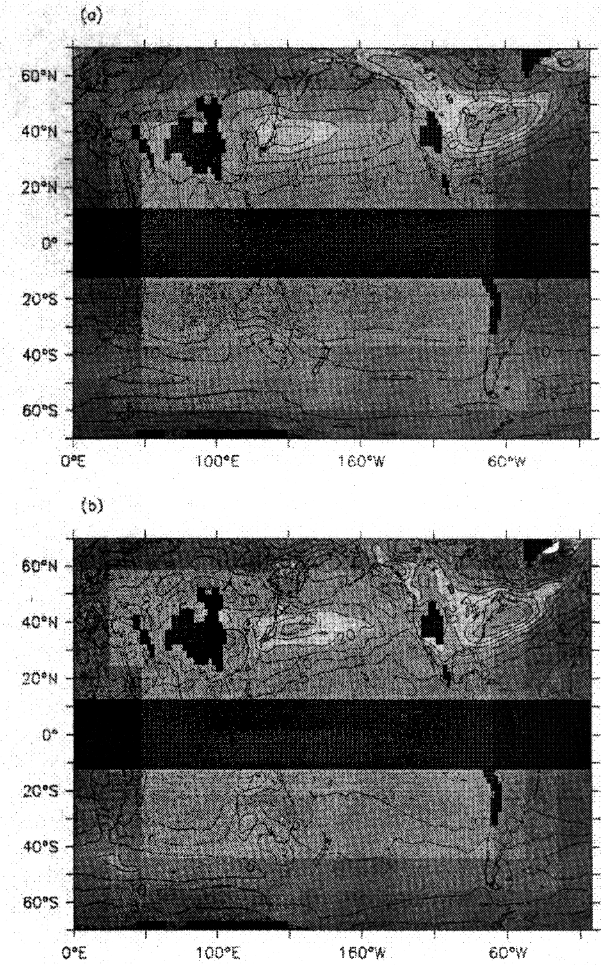

Figure 3. a) Meridional component of the divergent part of the DJF 850 mb heat flux; contour interval: 5 mK/s; shading interval: 2.5 mK/s. b) As in a), for the heat-flux estimate with $\alpha = 0.34$.

the scaling factor is included to bring the quantity's magnitude in line with that shown in Figure 1. The two figures share similar gross features, both showing maxima over the eastern basins of the North Atlantic and North Pacific, and a decrease going from the mid-latitudes towards the tropics. Areas of poorer agreement tend to occur where the flux and the temperature-gradient magnitudes are small and noisy.

When the regression analysis is repeated with 1000 mb data, the quality of the fit is somewhat poorer, and the value of α smaller than for the 850 mb data (see Table 1). From (3), we see that such a change in α might be associated with a variety of factors: a change in the velocity, temperature or streamfunction variance; in the velocity-temperature correlation; or in the mean temperature gradient. A detailed analysis reveals that the decrease in α going from 850 mb to 1000 mb is mainly the result of a large decrease in the temperature variance, with the other factors changing relatively little.

The drop in the temperature variance in the boundary layer presumably reflects the effect of strong thermal damping there, which could significantly alter the character of the downgradient mixing. *Swanson and Pierrehumbert* [1997] describe storm-track heat transports in terms of turbulent diffusion generated by a stochastic velocity field in the presence of thermal damping. The key parameter in this description is the ratio of the autocorrelation time of the velocity field to the damping term — if this ratio is small, damping has little effect on the turbulent diffusion. If, on the other hand, it is large, the temperature variance and therefore the diffusivity are reduced. In the atmospheric boundary layer, this ratio is order one.

We have also repeated this analysis after filtering all fields to retain only those time scales traditionally associated with storm-track eddies (2–6 days). The strength of the correlation between estimated and observed heat fluxes are similar, but with smaller values of α. For example, for the DJF 850 mb eddies, $\alpha \approx 0.25$.

Discussion

We have confirmed that the horizontal structure of the near-surface height variance leads to reliable estimates of the structure of the atmospheric near-surface horizontal heat flux. We have found that the scaling factor α is fairly uniform at a given height, but that it decreases going from the free lower troposphere to the boundary layer. We interpret this reduction in α as resulting from a thermal damping time that is comparable to the eddy decorrelation time.

Although *Holloway* [1986] introduced this method in order to obtain vertically integrated heat-flux estimates, relating eddy statistics in the interior to those at the surface is a separate problem that we do not address here. We view the *near-surface* horizontal diffusivity as an important quantity in its own right. Quasi-geostrophic theory teaches us that a theory for the effects of mesoscale eddies on the mean density structure of the ocean will involve theories for both the flux of potential vorticity within isopycnal layers and the *horizontal* flux of buoyancy near the surface (and ocean bottom). Estimates of the sea-surface horizontal diffusivity directly provide one of the pieces of information needed. But

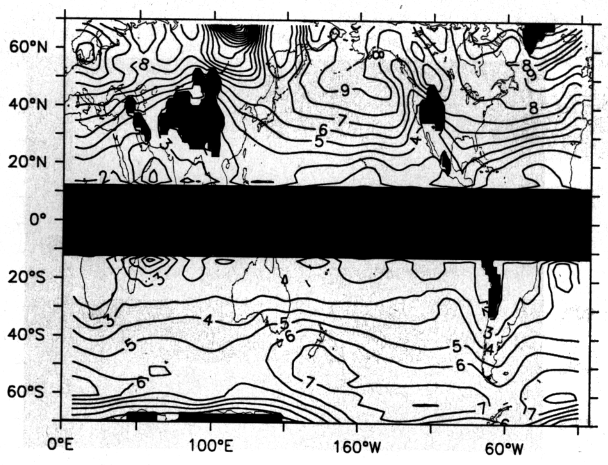

Figure 4. Ratio of the magnitude of the divergent part of the heat flux to the magnitude of the temperature gradient, divided by $\alpha_{DJF} = 0.34$, for the 850 mb data; contour interval: 10^6 m^2/s. The magnitudes have been smoothed using a seven-point box smoother.

in addition, the eddy fluxes of PV in the interior are closely related to the surface flux of buoyancy, if the fluxes are a consequence of baroclinic eddy production, so surface diffusivities provide constraints on interior diffusivities as well. See *Treguier et al.* [1997] for more discussion of these issues.

The results are relevant to the oceanic eddies to the extent that oceanic and atmospheric baroclinic geostrophic turbulence are similar. We see no clear reason to believe that oceanic and atmospheric geostrophic turbulence are fundamentally different in this regard. If anything, to the extent that the ocean mixed layer is less strongly damped than the atmospheric boundary layer, as measured by the ratio of damping time to the velocity autocorrelation time, there should be less concern than in the atmosphere as to changes in diffusivity near the surface. Analysis of high-resolution ocean models are needed to test this claim.

These results are not a parameterization of eddy transports, since we lack a theory that relates the diffusivity, represented by the r.m.s. eddy streamfunction, to the mean temperature distribution. Nevertheless, these results clearly suggest that the diffusivity coefficients used in the parameterization of eddies in ocean climate models must be strongly inhomogeneous in the horizontal in order to match the inhomogeneity of the height variability seen in the altimeter data [*Stammer*, 1997; *Treguier et al.*, 1997]. On an optimistic note, these results also imply that high-resolution ocean models that have been shown to reproduce the observed sea-surface height variability well (as in *Semtner and Chervin* [1992]) should also represent the near-surface eddy buoyancy fluxes well.

In summary, we argue that the altimeter-generated maps of sea-level variability may very well provide reliable estimates of near-surface eddy diffusivity over the world ocean.

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