The Role of Analogy in George Gamow’s Derivation of Drop Energy

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This article examines the role of the liquid drop analogy in George Gamow’s theory of nuclear structure and his subsequent derivation of nuclear energy. It argues that the correspondences constituting the analogy served distinct but cooperative ends, requiring Gamow to posit a relatively simple nuclear geometry that set him apart from his contemporaries, mostly shell theorists, and led to his successful derivation of nuclear energy in the fall of 1928.

Analogy has long been a source of sustained scholarly inquiry within rhetorical studies of science. However, the prospect of continued success is hindered by the widespread assumption that analogy functions collectively in a given rhetorical situation. This coarse framing of analogy as an indivisible whole distances rhetorical analysis from the specific candidate claims delivered to the context of invention by the analogy, claims that may serve different suasory ends (Little, 2000). Such a macroscopic conception limits our ability to see the effects of analogy on mathematical activity, a necessary point of clarity if we are to more fully elucidate the ways in which analogy informs scientific theory at formalized levels. It also obscures the ways in which analogy constrains the rhetor’s own thinking and development of subsequent claims, tacitly advancing the Aristotelian image of rhetors strategically drawing from analogy only those comparisons that serve their rhetorical purposes (Leff, 2003). Put another way, our history of scholarship shows that we tend to leave analogies in their aggregate form, choosing as our unit of analysis the analogy itself rather than the specific points of comparison generated by the analogy and enlisted by the rhetor or the audience. It also shows that when we do acknowledge that enlistment, we tend to frame it as a decidedly voluntary act, conferring almost unmitigated agency on the rhetor (Campbell & Clark, 2005). I suggest that these tendencies have dislocated us from an important site of inquiry,
which explains our historical failure to examine the ways in which analogy informs the invention of mathematical terms as well as the sensibilities surrounding acceptable operations involving those terms in the codification of scientific theory. This is an important yet seemingly unrecognized consequence of what Dilip Gaonkar (1997, p. 33) calls the “thinness of rhetorical theory,” with increasing relevance for rhetoric of science scholarship in general and scholarship implicating the epistemic status of analogy in particular.

To illustrate these notions, consider the landmark work of John Angus Campbell on Charles Darwin (Campbell, 1974, 1975, 1986, 1989, 1995). Vis-à-vis analogy, Campbell’s argument is clear and convincing: It was in the comparison to domestic breeding that Darwin found an effective vehicle for introducing natural selection within the norms and expectations of his Victorian audience. As we consider ways to build upon Campbell’s finding, however, we might ask ourselves the following: Which of Darwin’s claims were sponsored by the domestic breeding analogy? In what ways did the claims serve as an asset to Darwin’s argument? In what ways did they serve as a liability? In what ways did Darwin’s commitment to the analogy preclude, on formal or nonformal grounds, his enlistment of other, potentially suasory claims? In this new light, the coarse framing of analogy that served Campbell so well, that has served the rhetorical tradition so well—analogy as an indivisible whole operating collectively within a given situation—fails. More specifically, it fails to provide a sufficiently resolved analytic lens and attendant vocabulary for further effective inquiry.

Likewise, in The Rhetoric of Science, Alan Gross (1996, p. 27) recognizes Leonhard Euler’s “daring leap by means of analogy” to solve for the sum of the infinite series of squares. Gross also summarizes Erwin Schrödinger’s genetic code analogy, realized in James Watson and Francis Crick’s 1953 discovery of the structure of DNA, and describes how the comparison of codons to words led Crick and company astray in their search for a comma-free code (p. 29). Owing to his macroscopic conception of analogy, however, Gross does not, perhaps cannot, disassemble the analogies in question to better understand the effects of their constituent comparisons on the specific claims, terms, or mathematical operations made within the context of invention. His unit of analysis allows us to recognize the presence of analogy in Euler’s “applying the rules made for the finite to the infinite” (p. 28), but it fails to illuminate paths of further inquiry into the rules themselves or Euler’s novel application of them. Given his more immediate interest in a broader point about disciplinary differences in scholarly communication, Gross may not have been interested in a closer examination of the analogies in question. Nonetheless, the precedent he set is significant: His influential example affirms that it is within the conventions of strong rhetorical scholarship to acknowledge the “daring” and “imaginative leaps” of analogical reasoning without addressing the specific rhetorical options rendered probable or improbable by them.
More recently, Ken Baake’s (2003) *Metaphor and Knowledge* offers an impressive book-length analysis of another mode of comparison—metaphor—with particular emphasis on its epistemic role in scientific theory and mathematics. Of the relationship of metaphor to analogy, Baake is clear. After drawing from Max Black to articulate notable differences among metaphors, models, and analogies, he sets the stage for his chapter on the theory-constitutive role of metaphor with the following clarification: “Successful metaphors in science . . . form the primary basis of comparison. From these, more elaborate analogical connections can be extended” (p. 78). In the ensuing chapter, however, Baake does not examine the analogical connections extending from the metaphors in question. In a deft discussion of the relation of mathematics to science, he acknowledges the “deep structural similarities” that exist between systems of mathematics that invoke imaginary numbers (Colyvan as cited in Baake, 2003, p. 93); however, he stops short of investigating those structural similarities—likely, analogy by another name—and their effect on the framing of mathematical expression. (For a comprehensive criticism of the coarse framing of analogy in rhetoric of science scholarship, see Little, 2000.)

As I have suggested elsewhere (Little, 2000, 2001), Dedre Gentner’s (1983) structure mapping theory of analogy offers a promising avenue for further inquiry into the role of analogy in science. Prominent among cognitive psychologists, Gentner’s theory frames analogical reasoning as a process of mapping structurally related elements across two domains, the base and the target. When mapped, each element forms a structural “correspondence” between the domains—Gentner’s term for each point of comparison—and it is this correspondence that serves as the unit of analysis in her work. What is more, each correspondence is understood as having its own rhetorical effect by way of the candidate claims it sponsors. For example, a nuclear physicist reasoning via a planetary analogy would likely map the correspondence REVOLVES AROUND (X, Y) to the target domain, sponsoring candidate claims about angular momentum, gravitational attraction, orbital radii, and the types of mathematics deemed relevant to the problem. It is this correspondence-claim relationship that macroscopic conceptions of analogy miss.

Although isolated attributes such as RED (X) or HEAVY (X) can be mapped, Gentner claims that it is relations between objects (e.g., ATTRACTS [X, Y]) and higher-order relations between relations (e.g., CAUSE [ATTRACTS {X, Y}, COLLIDE {A, B}]) that people tend to compare in the process of analogical reasoning, providing a sort of deep structural alignment between the two domains (Gentner & Gentner, 1983; Markman & Gentner, 1993; Clement & Gentner, 1991). “In processing analogy,” argue Gentner and Jeziorski (1993, p. 448), “people implicitly focus on certain kinds of commonalities and ignore others.” While it is true, as Keith Gibson (2008) reminds us, that we have no ultimate assurance that an audience will interpret an analogy precisely along the lines of our intent, people do tend to have a shared sense of which commonalities to attend to and which ones
to neglect, the interpretive misfires in the history of science notwithstanding. There is, in other words, an overall orthos logos of the moment when it comes to analogical reasoning (Little, 2001): Not only can we expect the nuclear physicist to map the REVOLVES AROUND (X, Y) correspondence, we can also expect an audience of contemporary nuclear physicists to do the same. To borrow from the language of North American genre theory, analogical reasoning, especially within disciplinary discourse, is largely typified. It is in this sense that Gentner’s theory acknowledges the potentially constraining role of analogy in invention: From Gentner’s perspective, rhetors are not at liberty to draw selectively from analogy only those correspondences that serve their rhetorical purposes. Instead, because their audience will likely call to mind a common set of correspondences implicated by the analogy, rhetors must address those correspondences, regardless of whether they help or hinder their argument, or risk having the appropriateness of their reasoning called into question. It is not that the rhetor is held captive by the correspondences; rather, the notion that the audience will likely agree on the relative importance of various correspondences simply adds a hitherto unrecognized contour to the rhetorical situation. In short, Gentner realizes that the audience can “push back,” contra the classical model inherited from Aristotle. As such, her theory balances the role of agency across the rhetor, the text, and the audience, working toward the sort of “productive tension” advocated by Campbell and Clark (2005).

Recently, Heather Brodie Graves has taken advantage of the kind of correspondence-level rhetorical analysis encouraged by Gentner’s theory. In her insightful *Rhetoric in(to) Science*, Graves (2005, pp. 81–141) disassembles four analogies at the core of a group of solid-state physicists’ work with thin films to show how the individual correspondences not only substantively shaped their interpretation of empirical data but also led to further claims about the material properties of the films and the effect of the experimental practices on them. In one of the cases, she identified in the discourse of her participants an analogy between hoodoos, natural pillars of rock shaped by erosion, and the cross section of a superlattice, which invoked larger comparisons between natural erosion and ion beam etching. Unlike Campbell and Gross, Graves further specified the correspondences enlisted by the physicists—comparisons of CURVED SURFACE, LAYERED STRUCTURE, WORN AWAY SURFACE, VARIED HARDNESS, and VARIED DENSITY—and showed how the prospect of varied density led one physicist to reinterpret a problematic empirical finding as the result of “the ion beam etching the layers of the film at different rates, which he finally identified as related to their [different] densities—the insight that arose out of the hoodoo analogy” (p. 99). Their leaps were not as daring as Euler’s was, but Graves, in choosing as her unit of analysis the analogical correspondence over the analogy, was able to elucidate those leaps more directly and with more analytic clarity than any of her predecessors.

Likewise, Gibson (2008) offers a brilliant analysis of the four analogies driving Rodney Brooks’s artificial intelligence (AI) classic “Intelligence Without Repre-
presentation.” Taking his lead from John Stuart Mill and Edward Corbett, Gibson frames analogy as an ensemble of comparisons of “specific characteristics of otherwise distinct conceptualizations in service of a further comparison,” and therefore has in mind not the analogy per se as his unit of analysis but the specific points of comparison that span the two domains. Accordingly, Gibson takes the time to explicate the comparisons likely to have been activated by the analogies in the minds of the audience, comparisons that frame top-down AI researchers, connectionists, and the evolution of human intelligence in strategic ways and always in service to Brooks’s larger purpose. His study illustrates that it is at the level of the analogical comparison, what Gentner calls the “correspondence,” that the rhetorical dimensions of Brooks’s analogies come into analytic focus.

In this paper, I extend this Gentnerian style of analysis into the world of nuclear physics by examining the role of the liquid drop analogy in George Gamow’s derivation of nuclear “drop” energy.

First, I argue that the liquid drop analogy was comprised of three correspondences. Because the initial correspondence entailed the other two, Gamow was not at liberty to draw selectively from the analogy to serve his rhetorical purposes. Instead, it was an all-or-nothing subscription to the analogy and therefore to the likely candidate claims drawn from the three correspondences. Second, I argue that each correspondence served a distinct role in Gamow’s theorizing, from requiring him to posit a relatively simple nuclear geometry involving particles separated by a constant distance and held in place by a constant binding energy, to providing an analogical basis for framing the nucleus as a collection of identical particles, to requiring him to think of the nucleus as having two distinct regions of fundamentally different properties. Aside from setting him apart from many of his contemporaries, the analogy lent Gamow a conceptual simplicity that affected his mathematical treatment of individual nuclear particles as well as their relation to aggregated nuclei. Therefore, last, I argue that the recognition of the effect of the liquid drop analogy on Gamow’s energy derivation—in the invention of expressions such as \( AN_a^{(n+2)/3}/r_0^{n+2} \)—offers palpable evidence of the epistemic role of analogical reasoning at the most fundamental levels of scientific activity.

**INVENTING THE LIQUID DROP ANALOGY**

The fundamental reformulation of physics inaugurated by Max Planck (1901) and Albert Einstein (1905) brought about not only a wave of intellectual fervor unparalleled since the days of Newton but also a cloud of sustained confusion, if not despondency, among even the most capable physicists of the time. Gamow (1966, p. 17) called Planck’s quantum of energy “grotesque”; it was, in fact, a conceptual aberration that Planck himself had difficulty accepting. Although Einstein lauded Louis de Broglie’s (1924) wave interpretation of the electron as “a first feeble ray
of light on this worst of our physics enigmas” (qtd. in Pais, 1988, p. 252), Graves (1995, p. 106) reminds us of the “frustration and incredulity” that accompanied first-rate physicists, such as Heisenberg, as they attempted to reconcile the wave-particle duality of light, let alone that of matter. So profound were the initial difficulties of quantum theory that some physicists were willing to abandon the law of conservation of energy altogether if it meant they could avoid the implications of the Planck-Einstein worldview (Pais, 1988).

It was not until Erwin Schrödinger’s (1926) development of wave mechanics, Max Born’s (1926) probabilistic interpretation of wave functions, and Heisenberg’s (1927) introduction of the uncertainty principle that the cloud began to lift and quantum physics found a secure theoretical footing. The ascendancy of wave mechanics allowed physicists to return to differential equations as the mathematical tool of choice, relieved, for the most part, that the interlude of matrix mathematics required by Heisenberg’s earlier work had ended (Born, 1954; Pais, 1988, p. 255). To borrow from Thomas Kuhn (1962), the Fifth Solvay Conference, held in October of 1927, signaled a return to normal science, and physicists gathered in Copenhagen, Cambridge, Munich, and Göttingen to apply the new science of quantum mechanics to the complexities of the atom.

Into this fray arrived George Gamow in 1928, a graduate student at the University of Leningrad who, upon the recommendation of several senior physicists, had received permission from Moscow to spend the summer in Max Born’s institute in Göttingen. In his candid autobiography, My World Line, Gamow (1970, p. 58) described Göttingen as “a charming little city with an old and famous university. At that time, in the field of theoretical physics it could compete even with Copenhagen. It was buzzing with excitement caused by wave and matrix mechanics, which had been developed only two years before my arrival there.” Yet revising atomic theory in light of quantum mechanics was too popular and too frenetic an endeavor for Gamow’s liking, one that demanded considerable facility in the one area of mathematics he explicitly abjured: differential equations. “Thus,” he explained, “while all the quantum physicists in the world were attacking atoms and molecules, I decided to see what the new quantum theory could do in the case of the atomic nucleus” (1970, p. 59).

At the time, nuclear physics was a relatively dormant field—Ernest Rutherford (1911) having discovered the nucleus only 17 years earlier—and thus offered a modest empirical basis for Gamow’s theorizing. For over a decade, physicists had alluded to a new fundamental force—something other than the electromagnetic and the gravitational—to explain the intense attraction that held the particles of the nucleus together, but little progress had been made. What is more, the nucleus was thought to be comprised of protons and electrons that tended to aggregate into positively charged alpha particles, He^2+ (van den Broek, 1913; Rutherford, 1914). Of course, some lasting progress had been made. Abraham Pais (1988, p. 13) notes that by 1928 physicists well understood alpha and beta radioactivity as having nu-
clear origins, that is, that radioactivity resulted from nuclear rather than atomic dynamics. As well, they realized that hydrogen and helium nuclei were exceedingly small, their radii on the order of $10^{-12}$ centimeters (Pais, 1988, p. 230; Gamow, 1970, p. 59). However, nuclear physics, by all accounts, was still in its infancy.

Such was the setting for Gamow’s summer inquiry in Göttingen, where, after having read Rutherford’s (1927) latest work on the scattering of alpha particles, he envisioned immediately—“before I closed the magazine”—a quantum-mechanical explanation of alpha decay and set himself to the task of its full elaboration (Gamow, 1970, pp. 59–60). By the fall, Gamow had parlayed his Göttingen stay into a full academic year with Niels Bohr in Copenhagen, an appointment Bohr himself arranged at once after having met the young Ukrainian (Gamow, 1970, p. 64). It was during this time, in the distinct intellectual milieu of Göttingen or Copenhagen, that Gamow called to mind the surface forces acting on a drop of water while imagining the cohesive forces acting on a nucleus, and thus we see the genesis of the liquid drop analogy in the correspondence:

SURFACE TENSION (WATER DROP)
SURFACE TENSION (NUCLEUS)

That it was the idea of surface tension—and not, for example, the shared attribute of sphericity—that led Gamow to invent the liquid drop analogy: This is evident from the textual record. According to Roger Stuewer (1994, p. 79), Gamow believed that the liquid drop analogy probably occurred to him in conversation with Bohr, who, as a student in Copenhagen, had developed a method for determining the surface tension of water (Bohr, 1909, 1910). Stuewer (1994, p. 79) also suggests that the shape of the nuclear potential well in Gamow’s theory of alpha decay resembles “a volcanic cone containing energetic alpha-particles striking its interior wall,” which may have “sparked his thoughts” on the significance of the nuclear boundary. However, stronger and more direct support for the primacy of the SURFACE TENSION correspondence comes from Gamow’s own writing. In the opening of his 1929 talk before the Royal Society of London, and in his more fully developed paper of 1930, Gamow consistently identified the attractive force akin to surface tension as the salient feature of the analogy:

There is no contradiction between this and the Pauli-principle because alpha-particles having even charges must conform to Bose-Einstein statistics. Such an assembly of alpha-particles with attractive forces between them, which vary rapidly with the distance, may be treated somewhat as a small drop of water in which the particles are held together by surface tension. (Rutherford et al., 1929, p. 386)

In the discussion before the Royal Society on the constitution of the atomic nucleus held on February 7, 1929, I proposed a simple model of a nucleus built from al-
pha-particles in a way very similar to a water-drop held together by surface tension. (Gamow, 1930, p. 632)

Likewise, and more directly than in his previous writings, Gamow’s 1961 treatment of the topic in his popular Atom and Its Nucleus affirms that it was the problem of nuclear cohesion that led him to see in the surface tension of a water drop the beginnings of an analogical basis for his theory of nuclear structure. Under the heading “The Nucleus as a Fluid Droplet,” he (1961, pp. 100–101) focused on the attractive nuclear force, which leads seamlessly to the “familiar phenomenon of surface tension”:

In order to understand why the constituent parts of the nucleus stick closely together, we must assume that there exist between them forces of some kind, attractive in nature. . . . These forces that make them adhere, irrespective of the nature of the particles involved, are generally known as cohesive forces and are encountered, for example, in liquids, where they hold together the separate molecules and lead to the familiar phenomenon of surface tension. (his emphasis)

However, surface tension is not a force per se but a consequence—Gamow (1961, p. 100) called it an “effect”—of a relatively uniform distribution of particles with short-range attractive forces between them. Particles located deep within the distribution are surrounded by neighboring particles and thus held in place by the short-range attractive forces acting on them from all sides. Particles located at the surface, however, only have neighboring particles and short-range attractive forces on one side, the inward side. This effectively draws the surface particles inward, toward the center of the distribution, and it is this inward pull that is recognized as surface tension. Therefore, attendant to Gamow’s SURFACE TENSION correspondence are two others required for the effect:

UNIFORM DENSITY (WATER DROP)
UNIFORM DENSITY (NUCLEUS)
ATTRACTS AT SHORT RANGE (WATER MOLECULE, WATER MOLECULE)
ATTRACTS AT SHORT RANGE (NUCLEAR PARTICLE, NUCLEAR PARTICLE)

The relationship among these three correspondences illustrates an important point about agency. As I mentioned earlier, rhetoric of science scholarship has traditionally implied a rhetor who is able to draw strategically from analogy only those comparisons that serve the rhetorical purpose, a point made in more expansive form by Gaonkar (1997). Analogy is framed as a tool of sorts, deployed when useful, discarded when not. However, this view does not adequately capture the dynamics of Gamow’s rhetorical situation. Although the liquid drop analogy might have served him heuristically in many ways, enabling him to ac-
cept or reject various provisional claims along the way, and in some ways his au-
dience might have served a relatively passive role—though it is hard to think of
Rutherford or Bohr as in any way passive—one thing is clear: His enlistment of
the SURFACE TENSION correspondence logically demanded his enlistment of
the UNIFORM DENSITY and ATTRACTS AT SHORT RANGE correspon-
dences. Or, perhaps better articulated without the personification: If Gamow had
denied one of the attendant correspondences while still drawing from the notion
of surface tension, his audience would surely have rejected his argument on the
grounds of logical contradiction, the peculiarities of quantum mechanics not-
withstanding. Gamow did not have the luxury of full agency when elaborating on
the implications of the analogy. In their shared expectation of noncontradiction,
the audience potentially “pushed back,” constraining Gamow’s inventional op-
tions vis-à-vis the analogy. From a classical rhetorical perspective, we might say
that Gamow felt pressure to advance the implications of the analogy within the
strictures of the orthos logos of the moment, the shared sensibilities through
which audience members confer “right reasoning” in a given case (Aristotle,
1984a, 1984b). (In this particular case, Gamow would not have benefited from
selective enlistment; see Little, 2000, for a case where the rhetor’s inability to se-
lectively deny an analogical correspondence contributed to the ultimate demise
of the attendant theory.)

Gamow’s invention of these three correspondences, which form the liquid
drop analogy, set him apart from his colleagues, not in his appeal to analogy but
in his unique choice of base domain. At the time, in lieu of compelling empiri-
cal evidence, physicists had to assume some kind of attractive force to boot-
strap any theory of nuclear structure, to get any nuclear theory off the ground,
so to speak. Because so little was known about the nucleus, they necessarily
drew from other areas of science, reasoning via analogy from other base do-
 mains to the nucleus. Enamored by the success of planetary models in late-clas-
sical and early-quantum atomic physics (e.g., Nagaoka, 1904; Rutherford,
1911; Bohr, 1913), most nuclear physicists saw in the structure of the atom a
plausible basis for the structure of the nucleus. After all, the planetary analogy
had worked more than once and therefore might work again. As a result, shell or
satellite theories of nuclear structure emerged, which envisioned nuclear parti-
cles in shells or orbits about the nuclear center roughly akin to electrons about the
nucleus, with Rutherford’s (1919, 1927) satellite theory of the nucleus standing
as the most prominent and, according to Stuewer (1986, p. 154), “most thought-
ful” example of such a theory at the time. For whatever reason, Gamow resisted
the historical precedent found in the planetary analogy and chose not the atom as
his basis but the ostensibly unrelated water drop. This, as we will see in the next
section, benefited Gamow by offering him a substantially simplified conception
of the nucleus, which enabled him to achieve the first successful derivation of nu-
clear “drop” energy.
DERIVING DROP ENERGY

Analogy has long been understood as directing our attention and giving shape to our experiences (Perelman & Olbrechts-Tyteca, 1971; Graves, 1998, pp. 30–32; Gross, 1996). We can see this attenuating effect of analogy in, for example, Stephen Jay Gould and Richard Lewontin’s (1979) infamous “The Spandrels of San Marco and the Panglossian Paradigm.” Here, Gould and Lewontin use an analogy to distill the complex and varied positions of evolutionary biologists around the world into two, discrete camps along deeply asymmetrical lines, one associated with Voltaire, the other with feebleminded Pangloss (Bazerman, 1993). The analogy serves a sort of procrustean end: The multiplicity of opinions is suppressed, the value of spirited intellectual exchange neglected. What remains for readers of “Spandrels” are two positions, and only one of them, reductio ad absurdum, is tenable.

In the case of the liquid drop analogy, the UNIFORM DENSITY correspondence provided Gamow with an analogical basis for claiming that the distance between neighboring particles is roughly constant and thus that the binding energy per particle—that is, the amount of energy required to bind each particle to its neighbors—is constant as well. This deduction accorded exceedingly well with the recent empirical findings of Francis W. Aston (1927), who demonstrated that the binding energy per nuclear particle hovered around eight million electron volts for all but the lightest nuclei, a troubling prospect for shell theorists who expected nuclear particles to exhibit markedly different binding energies akin to those of electrons about the atom (Blatt & Weisskopf, 1991, pp. 3–7). Vital to Gamow’s purpose, the correspondence required a relatively simple geometry for the nucleus, one involving particles separated by a constant distance and held in place by a constant binding energy. It also enabled Gamow to conclude on analogical grounds that stable nuclei exist in static rather than dynamic equilibrium at a time when strong empirical research on the issue was not forthcoming.

The ATTRACTS AT SHORT RANGE correspondence further simplified Gamow’s conception of the nucleus, this time with respect to the particles themselves. Requiring only one type of nuclear constituent, the correspondence provided Gamow with an analogical basis for treating the nucleus as a collection of identical particles—alpha particles in his thinking at the time—exhibiting identical dynamics, akin to the molecules of a water drop. The fact that alpha particles were evenly charged, He\(^{2+}\), freed Gamow from what surely would have been the death knell of his theory: having to account for multiple quantum states within the nucleus, as the application of Wolfgang Pauli’s (1925) exclusion principle would have required. Comprised solely of evenly charged particles, Gamow’s nucleus obeyed Bose-Einstein rather than Fermi-Dirac statistics, and therefore was not governed by Pauli’s principle. Gamow therefore assumed that all alpha particles exist in their lowest energy state, quantum state 1. It was a point of pro-
found simplification that Gamow addressed early in his 1929 talk and in his 1930 paper:

There is no contradiction between this and the Pauli-principle because alpha-particles having even charges must conform to Bose-Einstein statistics. (Rutherford et al., 1929, p. 386)

The important point for the nuclear drop-model is the question of the quantum number to be ascribed to the different alpha-particles in the drop. The solution of this question is very simple; all alpha-particles in the nucleus must be considered to be in the same state with quantum number unity. This is due to the fact that the Pauli principle, which requires the electrons in an atom to be distributed in different shells, is not applicable to alpha-particles since they carry an even charge.

It can be shown from general principles of wave mechanics that in considering a collection of similar particles, each constructed from a certain number of protons and electrons, the Pauli principle must be applied to these particles only in the case when the total number of protons and electrons in each is an odd number, or in other words when the resultant charge of these particles is odd. (Gamow, 1930, p. 634; his emphasis)

In short, the correspondence enabled Gamow to universalize the nuclear particle and therefore typify his treatment of it, regardless of the size of the nucleus in question or the position of the particle within it. Such an invariant treatment of nuclear particles—particles of constant charge, mass, and quantum state—stood in stark contrast to shell theorists, whose analogies required them to consider the quantized orbit of the particle and thus the size of the nucleus in most of their calculations (Rutherford, 1927). Thus, we see in the opening of his 1930 paper, Gamow’s (p. 632) universal framing of the mutual potential energy of two alpha particles—any two alpha particles located anywhere within any nucleus—solely in terms of electrostatic repulsion \((+4e^2/r)\) countered by the short-range attractive force \((f[r])\) acting within the nucleus:

To begin with we shall treat a nucleus built from a certain number of alpha-particles only. To explain the possibility of a [stable] configuration of positively charged alpha-particles we must assume some attractive forces, which come into play only for a close approach of two alpha-particles and overbalance at short distances the forces due to electrostatic repulsion. . . . Although the law of these forces is not yet exactly determined, we can write the mutual potential energy of two alpha-particles at a distance \(r\) apart in the form

\[
U(r) = +4e^2/r - f(r)
\]

Practically, the conceptual simplicity lent to the nucleus by the analogy enabled Gamow to shuttle back and forth between expressions pertaining to a single alpha particle and expressions pertaining to an entire nucleus by simply multiplying or
dividing by $N_a$, the number of alpha particles in the nucleus, otherwise known as the atomic weight. Accordingly, in his derivation of nuclear energy, Gamow is able to move in one step from the internal energy of a single alpha particle

$$u + \kappa \sim - \frac{\hbar^2}{4mr_0^2}$$

to the total internal energy of a nucleus

$$E_a \sim N_a \cdot (u + \kappa) \sim - \frac{\hbar^2 N_a}{4mr_0^2}$$

This operation would not have been perceived as permissible by Gamow’s peers if he had not been able to assume the uniform density of the nucleus as well as its alpha particle conception, two assumptions warranted by the analogy.

Next, Gamow needed to express $E_a$ solely in terms of the variable $N_a$, which required his elimination of $r_0$ from the equation above. He accomplished this by deriving the “drop-equilibrium” of the nucleus, which showed that $r_0$ was proportional to the cube root of $N_a$. It is in this derivation that the epistemic role of the SURFACE TENSION correspondence becomes clear. George Lakoff and Mark Johnson (1980) discuss at length the role of everyday metaphors in the formation of ontological commitments. “Understanding our experiences in terms of objects and substances,” explain Lakoff and Johnson (1980, p. 25), “allows us to pick out parts of our experience and treat them as discrete entities or substances of a uniform kind.” Thus, “[a] clearing in the woods is seen as having a bounding surface, and we can view ourselves as being in the clearing or out of the clearing, in the woods or out of the woods” (p. 29; their emphasis). Although they are referring to metaphor, Lakoff and Johnson describe precisely the effect of the SURFACE TENSION correspondence on Gamow’s work: It required him to think of the nucleus as having only two distinct regions—the inner volume and the surface area—which he treated as generic, bounded regions independent of the individual properties of the nuclei in question. This set him apart from shell theorists of the time, whose planetary analogies conceptualized the nucleus in far more complex ways owing to the varied quantized orbits available to nuclei. Whereas shell theorists’ conceptual accounts of uranium were more complex than, for example, their accounts of helium, Gamow’s conception—which assumed static equilibrium and therefore was free of the notion of quantized orbits altogether—was identical in both cases, identical in fact across the entire periodic table of elements on this point. For Gamow, uranium and helium were conceptually identical—both have an inner volume and a surface area.

This dichotomy directed Gamow’s mathematical approach to the problem of drop-equilibrium, which he could now frame as the point where the outward force of the inner volume equals the inward force of surface tension (1930, pp. 633–635). Drawing from the work of Peter Debye, Gamow (p. 635) set his expres-
sion for the outward force \(h^2N_a/mr_0^5\) equal to his expression for the inward force \(AN_a^{(n+2)/3/r_0^{n+2}}\), resulting in the equation

\[
h^2N_a/mr_0^5 \sim AN_a^{(n+2)/3/r_0^{n+2}}
\]

which, in simplified form, he expressed as

\[
r_0 \sim R_0 \cdot N_a^{(n-1)/3 (n-3)}
\]

Having derived an equation for the radius of a nucleus \((r_0)\) in terms of atomic weight \((N_a)\)—an equation that accorded well with the recent empirical studies of light and heavy nuclei (p. 635)—he returned to his expression for the total internal energy of an alpha particle

\[
N_a \cdot (\mu + \kappa) \sim -h^2N_a^4/mr_0^2
\]

and, substituting his \(R_0 \cdot N_a^{(n-1)/3 (n-3)}\) for \(r_0\), expressed the total energy of any nucleus \((E_a)\) as a function of the cube root of the number of alpha particles within it \((N_a^{1/3})\), where \(h\) represents Planck’s constant, \(m\) the mass of a single alpha particle, and \(R_0\) an empirically determined coefficient of proportionality resulting from his elimination of \(r_0\):

\[
E_a \sim -h^2/4mR_0^2 \cdot N_a^{1/3}
\]

In other words, \(-h^2/4mR_0^2\) reduced to a known constant, leaving \(E_a\) varying simply with \(N_a^{1/3}\). Given \(N_a\), \(E_a\) can be easily determined, and vice versa.

Adding a final term to account for the electrostatic repulsion of the alpha particles at the surface, a factor that becomes increasingly relevant in heavy nuclei, he arrived at his final equation, the first successful derivation of nuclear drop energy:

\[
E_a \sim -h^2/4mR_0^2 \cdot N_a^{1/3} + (2e)^2/R_0 \cdot N_a^{5/3}
\]

The resulting graph of the alpha–drop energy curve accorded, argued Gamow (1930, p. 637), with Aston’s (1927) well-known mass defect measurements once he took free nuclear electrons into consideration, a prospect Gamow never fully accepted.

CONCLUSION

“We need a more sophisticated approach to the role of metaphor in technical writing,” wrote Jerome Bump in 1985, one that offers “an understanding not only
of the leading edge of research, represented by the discoveries of scientists like Bohr, Rutherford, Kelvin, and Maxwell, but also of the history and the most basic assumptions of science” (p. 448). Pursuing a more pedagogical line, Gibson (2008) has said as much of analogy: “[S]tudents will be better scientific communicators,” he argues, “if we can teach them to think specifically about the uses of inductive reasoning. . . . [W]e can, therefore, do our students a great service by including discussions of analogy, example, and metaphor in our classes” (my emphasis).

Dedre Gentner’s structure mapping theory of analogy offers a promising response to the epistemological and pedagogical projects raised by Bump and Gibson by framing analogy as an ensemble of correspondences, each capable of informing the rhetorical situation by way of the candidate claims it sponsors. This unit of analysis resolves the “thinness” problem identified by Gaonkar (1997) and provides a promising analytic lens for examining the specific ways in which analogy functions in science, including its largely neglected role in the invention of mathematical terms and the sensibilities surrounding the mathematical operations involving those terms in the context of a given rhetorical situation.

In this paper, I drew from Gentner’s theory to examine the role of the liquid drop analogy in George Gamow’s derivation of nuclear “drop” energy, a derivation that required a theory of nuclear structure at a time when empirical evidence was scant. I argue that the three correspondences comprising the analogy served Gamow in distinct but cooperative ways.

Latent in this work has been the notion of process in analogical reasoning in scientific theory construction, a notion that, to my knowledge, has yet to be acknowledged in the rhetoric of science literature. The analogy did not occur to Gamow fully formed. Rather, it was in the fall of 1928 that Gamow first called to mind the surface forces acting on a drop of water in conjunction with his work on the nucleus, thus setting in motion his subsequent development of the liquid drop analogy. The SURFACE TENSION correspondence required Gamow to think of the nucleus as having two distinct regions—the inner volume and the surface area—which he treated as generic, bounded regions independent of the individual properties of the nuclei in question. The correspondence also entailed two other comparisons, which, when Gamow realized them, simplified his conceptual and mathematical treatment of the nucleus in productive ways. The UNIFORM DENSITY correspondence required Gamow to assume a relatively simple nuclear geometry, one of equidistant particles held in static equilibrium by a constant binding energy, a prospect that accorded well with the recent empirical findings of Aston (1927). The ATTRACTS AT SHORT RANGE correspondence provided an analogical basis for claiming that the nucleus was comprised of one type of particle—alpha particles in Gamow’s thinking—which freed Gamow’s nucleus from the constraints of the Pauli exclusion principle, much to his satisfaction. Collectively, the three correspondences lent a conceptual simplicity to Gamow’s work that directed his
mathematical derivation of nuclear energy in important ways, most notably in providing him the logic for shuttling back and forth between expressions pertaining to a single alpha particle and those pertaining to an entire nucleus by multiplying or dividing by the atomic weight.

Also noticeable in this work has been the notion of *constraint*. Gamow’s commitment to the SURFACE TENSION correspondence demanded his acceptance of the UNIFORM DENSITY and ATTRACTS AT SHORT RANGE. Or, perhaps better put, his audience would have seen in the notion of surface tension the logical implication of uniform density and short-range attraction, just as Gamow did. In this sense, Gamow’s subscription to his initial insight constrained him in his inventional work: Out of bounds were candidate claims in contradiction to any of the three correspondences, which protected Gamow, in a sense, from serious consideration of the shell theories of his contemporaries. Put more generally, as I mentioned earlier, because their audience will likely call to mind a common set of correspondences implicated by the analogy, rhetors must address those correspondences, regardless of whether they help or hinder their argument, or risk having the appropriateness of their reasoning called into question. It is not that the rhetor is held captive by the correspondences; rather, the notion that the audience will likely agree on the relative importance of various correspondences simply adds a hitherto unrecognized contour to the rhetorical situation. In short, Gamow did not have the luxury of full agency when elaborating on the implications of the analogy. In their shared expectation of noncontradiction, the audience potentially “pushed back,” constraining Gamow’s inventional options vis-à-vis the analogy, a constraint that was much to his benefit.

The details of Gamow’s analogy become more valuable when situated within larger conversations of interest to the rhetoric of science. Important to the analogy-as-epistemic debate is the realization that mathematical terms such as $h^2N_a/mr_0^5$ and $AN_a^{(n+2)/3}/r_0^{n+2}$ are products of analogical reasoning. Invented by Gamow in the interplay between base and target domains, they gain the prospect of meaningful, reliable interpretation in theoretical nuclear physics in general and in Gamow’s derivation of nuclear energy in particular only by way of the assumptions afforded by the liquid drop analogy. No liquid drop analogy, no acceptable meanings available to these terms; and yet they form a considerable basis for Gamow’s “serious thought” and “plain reasoning” (Papin, 1992). Regardless of whether we see word meaning or mathematical expression as foundational to the precision of science—an interesting point raised by Baake (2003, pp. 105–107)—analogy is there, and in irreducible form. The terms $h^2N_a/mr_0^5$ and $AN_a^{(n+2)/3}/r_0^{n+2}$ are as real to Gamow as are his understandings of alpha particles and the charge on an electron, real in their consequence of leading to persuasive theory-data fit—which further problematizes the long-standing notion that analogy is merely nonformal accoutrement of proper modes of justification (Gross, 1996). This offers strong evidence for the epistemic role of analogy at the most fundamental lev-
els of scientific activity, from the early stages of theory construction to the final stages of refined mathematical codification.

Rhetorical studies informed by Gentner’s theory hold the potential to extend in important ways the line of inquiry at the intersection of rhetoric and mathematics begun two decades ago by Donald McCloskey (1985) and Philip Davis and Reuben Hersh (1987). Also important is the realization that the details of Gamow’s analogy could serve a more insightful role if situated within larger, correspondence-level comparative studies, studies capable of shining new light not only on the historical evolution of analogy in natural history and science (cf. Gentner & Jeziorski, 1993) but also on the personal, disciplinary, and cultural differences in analogical reasoning and their stabilized rhetorical implications for science (Little, 2001). Taking as its unit of analysis the correspondence rather than the analogy itself, Gentner’s theory offers a sufficiently granular analytic lens to detect variations in analogical reasoning that currently escape the notice of rhetorical analysis.

In broad strokes, Gamow’s work reveals the presence of analogy—performing constitutively, rigorously, and precisely—at the foundational levels of science, whereas Gentner’s work makes possible our understanding of the specific ways in which that performance is socially conditioned. This, as I see it, offers a valuable research agenda for scholars of analogy interested in contributing in palpable, practical ways to the larger disciplinary question of how the signs we choose affect the ways and things we can know.

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REFERENCES


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