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Cooperation, Solution Concepts and Long-term Dynamics in the Iterated Prisoner's Dilemma

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Abstract— **Solution concepts help designing co-evolutionary algorithms by interfacing search mechanisms and problems. This work analyses co-evolutionary dynamics by coupling the notion of solution concept with a Markov chain model of co-evolution. It is shown that once stationarity has been reached by the Markov chain, and given a particular solution concept of interest, the dynamics can be seen as a Bernoulli process describing how the algorithm visits solution and non-solution sets. A particular analysis is presented using the Iterated Prisoner's Dilemma. By numerically computing the Markov chain transition matrices and stationary distributions, a complex and strong relation between variation and selection is observed.**

I. Introduction

Solution concepts help designing co-evolutionary algorithms by interfacing co-evolutionary search mechanisms and problems [1]. For an arbitrary point in the search space, a given solution concept will determine whether it could be considered as a satisfactory solution or not. Several pathologies present in co-evolutionary algorithms are hypothesized to be caused by a lack of rigour in solution concepts [1]. Then, adequately coupling a *solution concept of interest* with a particular co-evolutionary search mechanism will possibly diminish such pathologies. Accordingly, we aim at characterising the parameter space of a co-evolutionary algorithm given a solution concept for the benchmark problem of the emergence of cooperation in the Iterated Prisoner's Dilemma.

It has been shown that under certain conditions, a class of co-evolutionary algorithms can be described as Markov chains. Using simple operators of mutation and selection, and a simple representation, the resulting Markov chain (MC) is ergodic. Ergodicity implies that the dynamics is described in the long-term by a stationary probability distribution on the space of populations [2], i.e., once stationarity has been reached, the dynamics is completely described by independent samples of such stationarity distribution.

In this work, the co-evolutionary dynamics is studied in the long-term by analysing the *solution visiting* process that results from describing the way the asymptotic dynamics of the MC visits the partition defined by a solution concept, which turns out to be a Bernoulli process. The idea behind this approach is to suggest a particular path towards the study of how adequately tunned combinations of co-evolutionary search mechanisms and solution concepts may improve the quality of solutions in certain problems. An analysis is presented studying the emergence of cooperation in the multi-level Iterated Prisoner's Dilemma.

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II. Preliminaries

A. Co-evolutionary search as a Markovian process

This section is based on the model presented by Liekens [2], which allows to view co-evolutionary dynamics as a MC. It uses a simple genetic representation and an evaluation scheme based on averaging two-player game results. In addition to a proportional selection operator, an elitist selection operator is introduced to analyse the effects of strong selection.

1) Individuals and Populations: Consider the set of possible strategies (individuals) $\mathbf{X} = \{1, 2, ..., n\}$ for an arbitrary two-player game described by a $n \times n$ matrix A. The space of possible populations of r individuals is thus given by the set $\mathbb{P}$ of multisets of size r, whose elements are in $\mathbf{X}$.

A population $\mathbf{x} = [x^{(1)}, x^{(2)}, ..., x^{(r)}]$, could be represented as a vector $\mathbf{p^x}$ of size n, in which the $j-th$ element, with $j \in \mathbf{X}$, is the proportion of elements of type j in population $\mathbf{x}$, denoted $p(j, \mathbf{x})$.

2) Evaluation: The fitness of an individual will be its average payoff from playing all the individuals in the same evolving population. Then, at generation t, the fitness of individual i in population $\mathbf{x}_{(t)}$ will be given by:

$$f(i, \mathbf{x}_{(t)}) = \sum_{j \in \mathbf{X}} A_{i,j} p(j, \mathbf{x}_{(t)}) \tag{1}$$

where $A_{i,j}$ is the payoff received by strategy i when it plays against strategy j.

3) Selection: Given $f(i, \mathbf{x}_{(t)})$, the probability of selecting individual i from population $\mathbf{x}_{(t)}$ is given by:

$$s_p(i, \mathbf{x}_{(t)}) = \frac{f(i, \mathbf{x}_{(t)}) p(i, \mathbf{x}_{(t)})}{\Sigma_{j \in \mathbf{X}} f(j, \mathbf{x}_{(t)}) p(j, \mathbf{x}_{(t)})}$$

This selection operator selects individuals in proportion to their fitness and number of copies in the population.

In addition, a strongly narrow elitist selection operator will be considered here. This turns out to be particularly suitable to have an insight into the role of elitism in co-evolutionary settings.

Let $M(\mathbf{x}_{(t)})$ denote the set of individuals with maximum fitness value in population $\mathbf{x}_{(t)}$, i.e.,

$$M(\mathbf{x}_{(t)}) = \{i \in \mathbf{x}_{(t)} | f(i, \mathbf{x}_{(t)}) = \max_{j \in \mathbf{x}_{(t)}} f(j, \mathbf{x}_{(t)})\} \quad (2)$$

Then, the probability of selecting individual i from population $\mathbf{x}_{(t)}$ is:

$$s_e(i, \mathbf{x}_{(t)}) = \frac{\mathbf{1}_{M(\mathbf{x}_{(t)})}(i)}{|M(\mathbf{x}_{(t)})|} \quad (3)$$

where $\mathbf{1}_{M(\mathbf{x}_{(t)})}(a)$ is 1 if a is in $\mathbf{x}_{(t)}$, and 0 otherwise.

4) Variation Operator: A simple variation operator is considered. An individual i will be mutated with probability μ into another individual j, which will be uniformly chosen from set $\mathbf{X} - \{i\}$, i.e., with probability $\frac{1}{n-1}$.

The operator described is a strong asexual variation operator. Notice that sexual reproduction (recombination) is not considered in this case.

5) Reproduction: Let $m(i, \mathbf{x}_{(t)})$ be the probability that an individual i becomes part of the next population, $\mathbf{x}_{(t+1)}$. If $s_*(i, \mathbf{x}_{(t)})$ denotes the selection probability associated to the selection operator to be used (either $s_e(i, \mathbf{x}_{(t)})$ or $s_p(i, \mathbf{x}_{(t)})$), then $m(i, \mathbf{x}_{(t)})$ can be expressed as:

$$\begin{aligned} m(i, \mathbf{x}_{(t)}) &= (1-\mu) s_*(i, \mathbf{x}_{(t)}) + \mu \frac{1}{n-1} \sum_{j \neq i} s_*(j, \mathbf{x}_{(t)}) \\ &= (1-\mu) s_*(i, \mathbf{x}_{(t)}) + \frac{\mu}{n-1}(1 - s_*(i, \mathbf{x}_{(t)})) \end{aligned} \quad (4)$$

6) Transition Matrix and Long-term dynamics: Since population size is r, the probability measure defined by $m(i, \mathbf{x}_{(t)})$ on $\mathbf{X}$ has to be sampled r times in order to get the new population. This sampling procedure is claimed to introduce drift effects that cannot be predicted using evolutionary game theory techniques [2]. In order to determine the probability that a population $\mathbf{x}_{(t+1)}$ be generated from a population $\mathbf{x}_{(t)}$, a multiniomial distribution is used. Thus,

$$Pr[\mathcal{E}(\mathbf{x}_{(t)}) = \mathbf{x}_{(t+1)}] = \frac{r!}{\prod_{i \in \mathbf{X}} (rp(i, \mathbf{x}_{(t+1)}))!} \prod_{i \in \mathbf{X}} m(i, \mathbf{x}_{(t)})^{rp(i, \mathbf{x}_{(t+1)})}, \quad (5)$$

Where $\mathcal{E}(\mathbf{x}_{(t)})$ is the operator representing the result of a step of the Markov chain when it is applied to the population $\mathbf{x}_{(t)}$.

Once $m(i, \mathbf{x}_{(t)})$ is completely identified, it can be shown that the MC defined by the probability above is ergodic whenever $\mu \neq 0$. Hence, an associated stationary distribution on populations describing the long-term dynamics of the system could be calculated.

B. Solution Concepts in Co-evolution

A solution concept is a binary partition of the search space that determines whether a particular point in the space is a solution or not, i.e., it partitions the search space into two classes: *solutions* and *non-solutions*. A solution concept is required regardless the domain or the representations involved in the problem [3], [1].

Solution concepts are inherent to search problems, but search mechanisms must implement them properly in order to succeed. Thus, solution concepts provide the bridge between search problems and search mechanisms.

In designing a co-evolutionary algorithm, it is important to consider whether the solution concept implemented by the algorithm (i.e., the set of individuals to which it may converge) corresponds with the intended solution concept [1]. It is argued that the cause of several pathologies found in co-evolutionary methods are due to a trivial approach to coupling solution concepts of interest with co-evolutionary search methods.

C. IPD and multi-level IPD

The abstract mathematical game known as the Prisoner's Dilemma (PD) has been widely studied in several disciplines to analyse the emergence of cooperative behaviour [4], [5]. In its simplest form the prisoner's dilemma is a two-choice game described by the following matrix:

TABLE I
PAYOFF MATRIX FOR THE PRISONER'S DILEMMA

		Player 1	
		Cooperate	**Defect**
Player 2	**Cooperate**	(R,R)	(S,T)
	Defect	(T,S)	(P,P)

where $T > R > P > S$ and $R > \frac{S+T}{2}$.

A dilemma arises in the game due to the fact that if both players were purely rational, they would never cooperate. In this work, the parameters of the game are chosen as $T = 5$, $R = 4$, $P = 1$ and $S = 0$.

In the multi-level PD, individuals have more than two strategies to play. This is intended towards modelling more accurate real-world dilemmas. The payoffs for the multi-level PD are a linear interpolation of the payoffs for the simple 2-choices PD. Here, strategies will be represented as real numbers in the interval $[-1, 1]$, where 1 represents full cooperation and -1 represents full defection. For instance, a 4-choice matrix (specifying payoffs for one player) is:

TABLE II
PAYOFF MATRIX FOR THE 4-LEVEL PRISONER'S DILEMMA

		Player 1			
		1	**1/3**	**-1/3**	**-1**
	1	4	$2\frac{2}{3}$	$1\frac{1}{3}$	0
Player 2	**1/3**	$4\frac{1}{3}$	3	$1\frac{2}{3}$	$\frac{1}{3}$
	-1/3	$4\frac{2}{3}$	$3\frac{1}{3}$	2	$\frac{2}{3}$
	-1	5	$3\frac{2}{3}$	$2\frac{1}{3}$	1

In the iterated Prisoner's Dilemma (IPD), the game described above is played several times, which allows agents to switch choices and thus, exhibit mutual cooperation or retaliation. The IPD game is widely regarded as a standard model for the evolution of cooperation, and constitutes a benchmark problem in the co-evolution of game strategies [6].

Co-evolution in the IPD occurs when the fitness of individuals is assessed by confronting them with other playing individuals. The fitness of an individual is usually the average payoff obtained by confronting other individuals in the same population. In this case, a dynamic landscape arises from strategic interaction.

In general, it has been shown that for co-evolution in the multiple-level IPD, introducing more choices reduces the frequency of mutual cooperation [7], [8]. While many works in this area evolve trigger strategies for the IPD (e.g, tit-for-tat-like strategies, rules, etc) [9], [10], in this work choices will be evolved, and the main aim is to produce populations in which cooperative individuals are abundant.

III. Solution Concepts imply a Bernoulli distribution in the long-term

When a solution concept is coupled to the stationary distribution that describes the asymptotic dynamics of co-evolutionary search, it defines a *solution visiting* process that shows how such dynamics visits solutions and non-solutions, as defined by the solution concept. Notice that once stationarity has been reached, and due to the ergodic nature of the Markov Chain, each population can be seen as an independent realisation of the stationary distribution, therefore, the solution visiting process can be seen as a series of independent Bernoulli trials (as it visits solutions and non-solutions).

Formally, let η be the stationary distribution on the population space $\mathbb{P}$, associated to the Markov chain defined by the search mechanism. Also, let $[\mathbb{P}_S, \mathbb{P}_N]$ be the partition of $\mathbb{P}$ defined by a solution concept, with $\mathbb{P}_S$ the solution set and $\mathbb{P}_N$ the non-solution set. Considering as *success* a visit to one of the solutions defined by the solution concept, then probabilities p and q, namely success and failure probabilities, are defined by

$$p = \sum_{\mathbf{x}\in\mathbb{P}_S} \eta(\mathbf{x}), \qquad and \qquad q = 1 - p; \tag{6}$$

thus, the independent process $Y_t, t =, 1, 2, ...$ with

$$Pr(Y_t = 1) = p \tag{7}$$

is the Bernoulli process that describes the solution visiting process in the long term.

The probability p, that defines this process changes when the parameters of the co-evolutionary process are modified. How p changes, when the parameters of co-evolutionary search are modified, will be the subject of study in next section.

IV. Results: The Iterated Prisoner's Dilemma With Multiple Levels of Cooperation

In this section, we will study how different search mechanisms (i.e., co-evolutionary search with different levels of variation and different selection schemes) implement a family of solution concepts, strictly related to the emergence of cooperation in the IPD game. In the context of the Bernoulli process defined above, the probability of success induced by different parameters of the co-evolutionary algorithm will be calculated.

A. Level of cooperation Solution concept

In the context of the IPD, we are interested in finding populations in which cooperation is spread. Thus, it is important to emphasise that solutions will be collectives (instead of individuals), and then, we will define a solution concept on populations of strategies in the IPD game.

As mentioned before, individuals will be choices represented as numbers in the interval $[-1, 1]$, where -1 means full defection and 1 means full cooperation. Thus, given a population of strategies $\mathbf{x} = \{x^{(1)}, x^{(2)}, ..., x^{(r)}\}$, the level of cooperation of $\mathbf{x}$ is defined as:

$$L(\mathbf{x}) = \sum_{i=1}^{r} x^{(i)} \tag{8}$$

Notice that the level of cooperation of a population increases with the number of cooperative individuals in it. Then, for an arbitrary level of cooperation T, the *level of cooperation solution concept* is defined as:

$$\begin{aligned} \mathbb{P}_S &= \{\mathbf{x} | L(\mathbf{x}) \geq T\} \\ \mathbb{P}_N &= \{\mathbf{x} | L(\mathbf{x}) < T\} \end{aligned} \tag{9}$$

Thus, a solution will be a population that satisfies a minimum cooperation threshold T.

B. Parameters

In this work, the classic IPD and the multiple level IPD with 3 and 4 levels of cooperation will be considered. The parameters used to generate the IPD games are the ones reported in section II-C. Thus, in the classic IPD game the set of choices is $\{-1, 1\}$, and for the 3-level and 4-level cases they are $\{-1, 0, 1\}$ and $\{-1, -\frac{1}{3}, \frac{1}{3}, 1\}$ respectively.

The transition matrix (as described above) associated to different levels of mutation, selection mechanisms (either elitist or proportional), and population sizes of 10 and 20 individuals will be calculated using equation 5. Another parameter will be the solution concept, defined by a minimum level of satisfactory cooperation T (see eq. 9). Notice that minimum levels of cooperation satisfy $-r \leqslant T \leqslant r$; a population with level r is completely cooperative while a population with level $-r$ is completely defective.

The results will be presented as follows. Given a population size r, an IPD game, a selection mechanism and a mutation rate; the transition matrix that describes co-evolutionary dynamics is calculated using equation 5. Then, the associated stationary distribution is numerically computed, and p is calculated for all the possible values of T (using eq. 6).

C. Results

The graphics below show how different parameters such as mutation rate, IPD game, population size, selection mechanism, and minimum satisfactory level of cooperation impact p, i.e., the probability of success associated to a Bernoulli process once stationarity has been reached in the long-term.

Each curve shows how well a co-evolutionary algorithm performs for a particular solution concept in the long-term, when the mutation rate is changed using a fixed selection scheme.

1) General Observations: Each solution concept can be interpreted as a measure of how hard the problem is, i.e., the greater the cooperation threshold is, the harder it is to find a satisfactory population. Notice that there is always a trivial solution concept, when T equals $-r$, for which all the populations are cooperative enough, thus a solution is reached in the long-term with probability p equal to 1.

Notice that due to the nature of solution concepts defined in this work, p is always a positive probability, since there is at least one population in $\mathbb{P}$ whose level of cooperation is T. This feature highlights the need for coupling solution concepts and search mechanisms: while solution concepts define *what to look for*, search mechanisms defines *where to look*.

Note that when proportional selection is used (see figures 1, 3, 5, 7 and 9), if p is seen as a function of μ it appears to be monotonically increasing for large values of T, i.e., the harder the problem is, the greater variation should be introduced in order to gain a significant probability of success. From the graphics, it is also clear that as the cooperation threshold increases, it becomes less likely to find mutually cooperating populations.

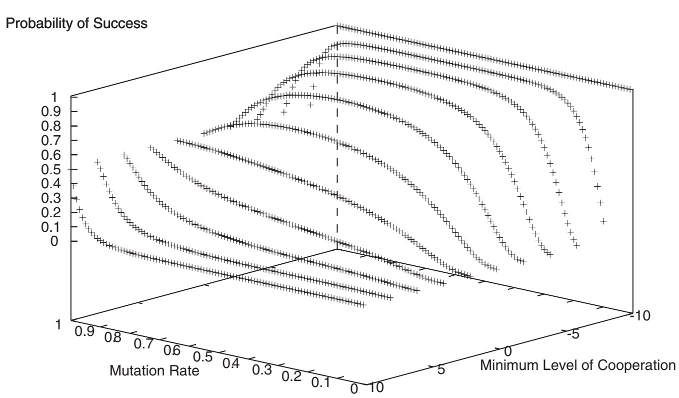

Fig. 1. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 2-Level IPD with 10 individuals and proportional selection

2) Role of Mutation and Selection: It can be observed in the graphics that high mutation tends to be beneficial in general. However, when elitist selection is used (see figures 2, 4, 6, 8 and 10), there appears to be an intermediate optimum level of mutation for which p is maximum. This does not

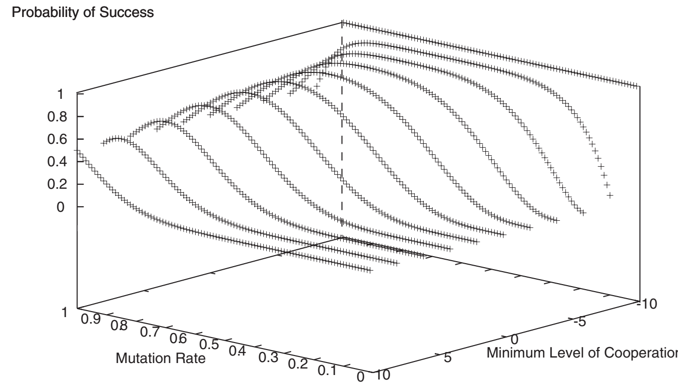

Fig. 2. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 2-Level IPD with 10 individuals and elitist selection

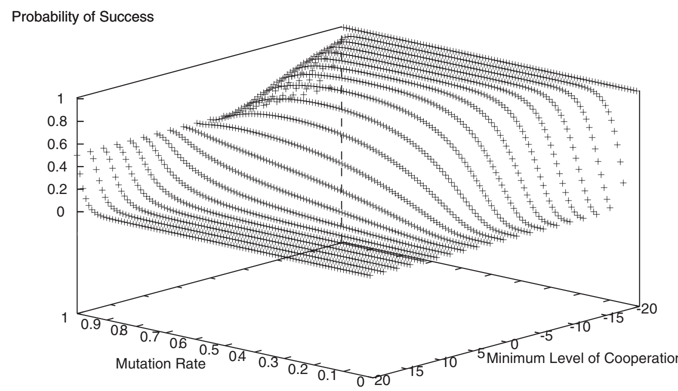

Fig. 3. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 2-Level IPD with 20 individuals and proportional selection

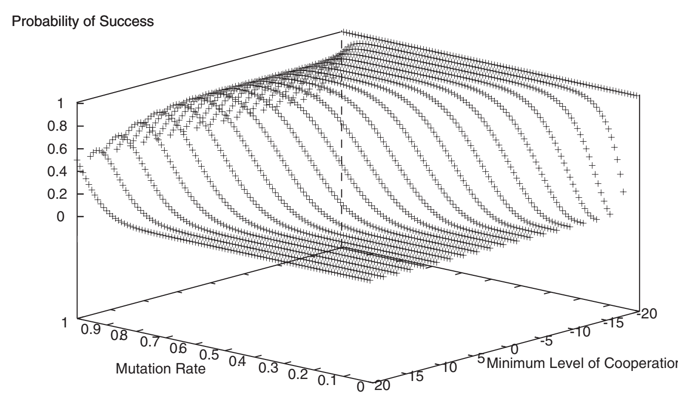

Fig. 4. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 2-Level IPD with 20 individuals and elitist selection

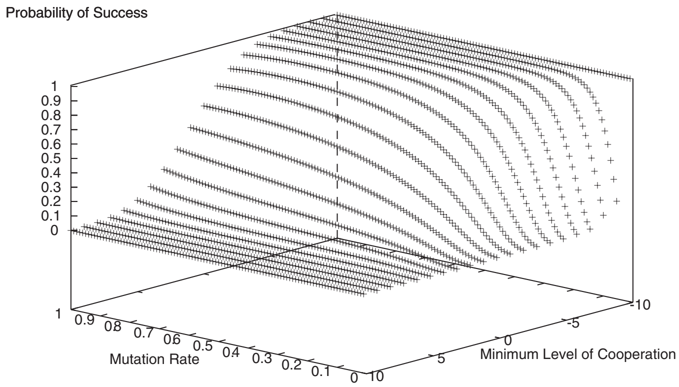

Fig. 5. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 3-Level IPD with 10 individuals and proportional selection

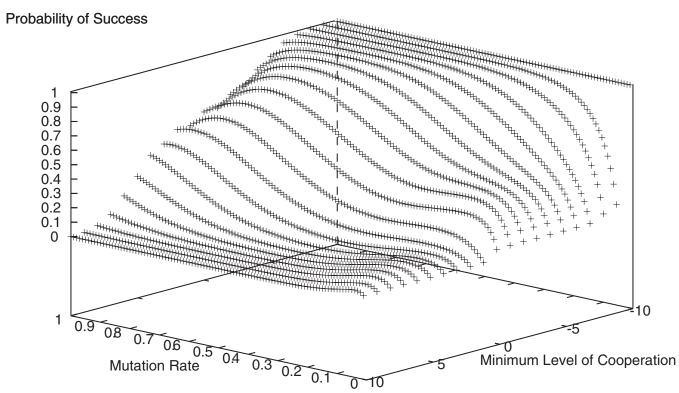

Fig. 6. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 3-Level IPD with 10 individuals and elitist selection

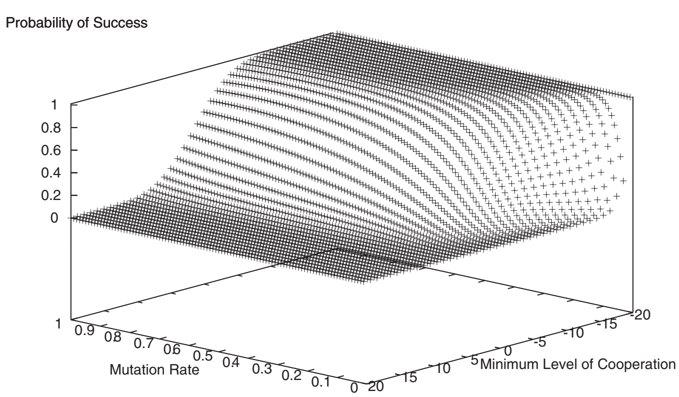

Fig. 7. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 3-Level IPD with 20 individuals and proportional selection

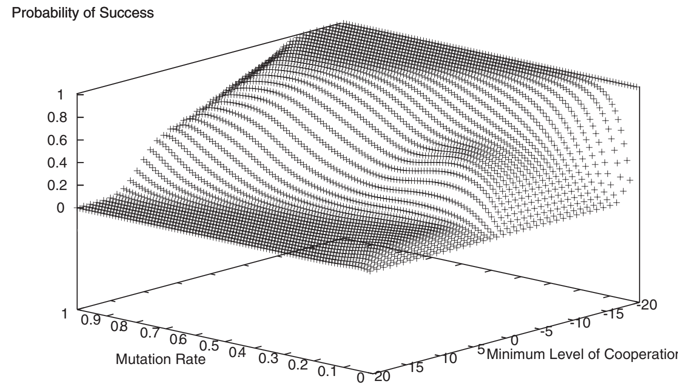

Fig. 8. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 3-Level IPD with 20 individuals and elitist selection

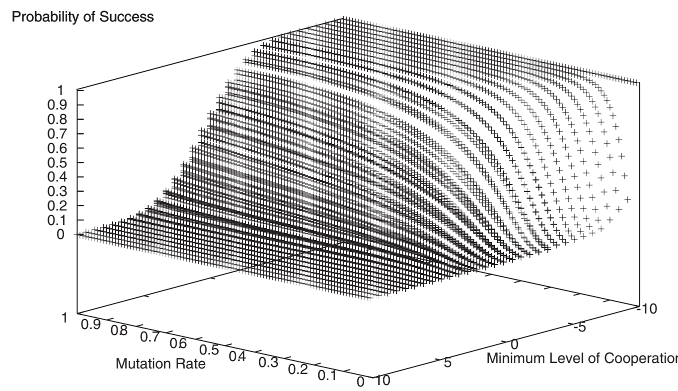

Fig. 9. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 4-Level IPD with 10 individuals and proportional selection

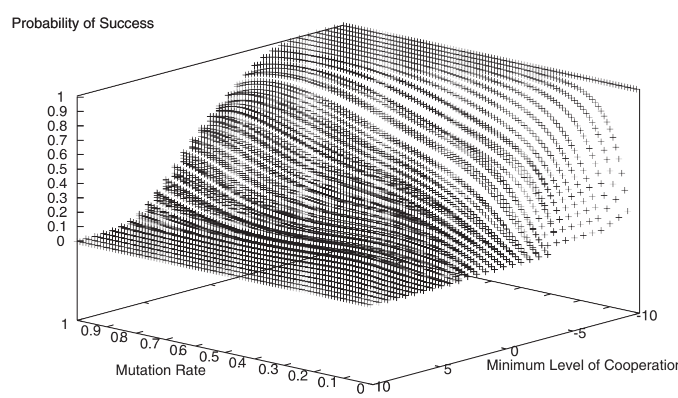

Fig. 10. Probability of success for multiple mutation rates and solution concepts defined by a minimum level of cooperation using 4-Level IPD with 10 individuals and elitist selection

hold for proportional selection cases, in which maximum p appears when mutation is the maximum value. This effect is more evident as the difficulty of the problem, posed by the solution concept, increases.

On the other hand, it can also be observed that elitism is beneficial specially in hard problems. Notice that when proportional selection is used, values of p decrease faster for increasing minimum levels of cooperation. This effect appears to be more evident as more choices are introduced.

3) Impact of the number of choices: It can be observed that as more choices are introduced, less mutual cooperation is likely to evolve in the long term. Note, for instance, that for 2-level games (figures 1, 2, 3 and 4) and high mutation long-term Bernoulli trials approach a flipping coin. This is considerably different from what happens for 3 and 4-level games (figures 5, 6, 7, 8, 9 and 10), when probability of success p is actually increased with the help of a strong (elitist) selection.

4) Impact of population size: Since only two values of population size were considered, there is no enough evidence at this point to formulate any hypotheses with regard to this parameter. Exploring more values of population sizes would require considerable more computational power. This remains to be a very important parameter, since it is hypothesized to introduce important drift effects [11]. For the values studied here, shapes of the graphics are somehow preserved when fixing the other parameters.

V. Conclusion and further work

The main result of this work is the introduction of a Bernoulli visiting process tied to a given solution concept, as a way to gain insight into how co-evolutionary search parameters may be more or less adequate in implementing certain solution concepts in the long-term.

A preliminary analysis is presented using the benchmark problem of the IPD. Although other studies use different representations and models, general empirical suggestions in other works are verified [10], [9]. Particularly, in regard to the negative effects of several choices when it comes to finding cooperative strategies, and the benefits of high values of variation as a useful mechanism to generate behavioural diversity.

It is arguable that an adequate combination of variation and selection forces may work out better for certain problems (solution concepts). In the light of a complex relationship between selection and variation, strongly mediated by solution concepts, the importance of introducing solution concepts into frameworks of dynamic analysis is emphasized.

It is important to keep in mind that the results of this work are limited to the long-term behaviour, since the analysis is performed once the MC has converged to stationarity.

However, in many practical applications the transient behaviour is important since usually the time a MC takes to reach stationarity is unknown (and long). As a further direction to gain insight into transient dynamics and convergence times, an approach similar to [12] is suggested.

Many questions remain open and several paths are suggested to continue this study. It is important to consider other games with different structures and motivations. Also, it is necessary to consider more solution concepts. The elements involved in this analysis can be extended and certainly enriched by incorporating formal tools of statistical analysis of Bernoulli processes (geometric distributions, MC burn-in times, etc). The results also suggest that it may be fruitful to work on dynamic adaption of selection and mutation schemes in co-evolution, since fixed values may not be enough for the rich dynamics introduced by co-evolutionary landscapes.

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