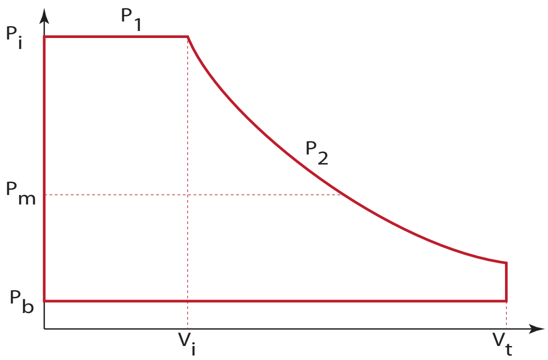
1 Problem



Calculate a rough estimate of the Mean Effective Pressure (MEP) of an ideal expanding steam cycle.

2 Definitions

2.1 Constants

- P_b : back pressure (PSIa)
- P_i : input pressure (PSIa)
- V_t : total displacement volume (in^3)
- V_i : volume at steam cutoff (in^3)
- R_e : ratio of expansion (dimensionless)
- a : expansion constant

2.2 Functions

- P_m : mean effective pressure (PSIa)
- P_1 : pressure during input (PSIa)
- P_2 : pressure during expansion (PSIa)

3 Solution

The pressure during input is assumed to be constant:

$$P_1 = P_i \tag{1}$$

The total and cutoff volumes are related to eachother by the ratio of expantion:

$$\begin{aligned} R_e &= \frac{V_t}{V_i} \\ V_i &= \frac{V_t}{R_e} \end{aligned} \tag{2}$$

The relation of pressure and volume during expansion can be idealized by the following equation:

$$\begin{aligned} PV = & \ a \\ a & = P_iV_i \end{aligned} \tag{3}$$

since we know the pressure and volume at the point of cutoff. This leads to the following formulation of the pressure during expansion:

$$\begin{aligned} P_2V &= a \\ P_2V &= P_iV_i \\ P_2 &= \frac{P_iV_i}{V} \end{aligned} \tag{4}$$

The MEP is equal to the area within the PV diagram, divided by the total volume that it spans:

$$P_m = \frac{\int_0^{V_t}\{P_1, P_2\} - P_b dV}{V_t} \tag{5}$$

$$\begin{aligned} P_m &= \frac{\int_0^{V_i} P_1 dV + \int_{V_i}^{V_t} P_2 dV - \int_0^{V_t} P_b dV}{V_t} \\ P_m &= \frac{\int_0^{V_i} P_i dV + \int_{V_i}^{V_t} \frac{P_iV_i}{V} dV - \int_0^{V_t} P_b dV}{V_t} \\ P_m &= \frac{P_i \int_0^{V_i} dV + P_iV_i \int_{V_i}^{V_t} \frac{1}{V} dV - P_b \int_0^{V_t} dV}{V_t} \\ P_m &= \frac{P_iV_i + P_iV_i[lnV_t - lnV_i] - P_bV_t}{V_t} \\ P_m &= \frac{P_iV_i[1 + lnV_t - lnV_i] - P_bV_t}{V_t} \\ P_m &= P_i\frac{V_i}{V_t}[1 + ln\frac{V_t}{V_i}] - P_b\frac{V_t}{V_t} \end{aligned}$$

$$P_m = P_i\frac{1 + lnR_e}{R_e} - P_b \tag{6}$$