Architecting the Very-Large-Aperture Flux-Pinned Space Telescope: A Scalable, Modular Optical Array with High Agility and Passively Stable Orbital Dynamics

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Direct detection of exoplanets and more detailed Earth observation demand space telescopes that are more powerful than those currently used. Our proposed modular design uses superconductive flux pinning instead of mechanical trusses to maintain the relative position and orientation of mirrors. Flux pinning requires no active control and only enough power to keep the superconductors below 88K. We discuss a sparse-aperture architecture in which identical, deformable mirror segments on flux-pinning modules lie on a spherical manifold surrounding a central detector. The resulting low-power telescope is isotropic, passively stable, scalable, agile, and easily repaired, and the field of regard is unlimited.

Nomenclature

\begin{align*}
\theta &= \text{angular resolution} \\
\vartheta &= \text{polar coordinate} \\
\varphi &= \text{angle of incidence} \\
\lambda &= \text{wavelength} \\
a &= \text{distance between the focus and the vertex of a parabola} \\
d &= \text{aperture diameter} \\
d_{cf} &= \text{distance between the point } (r_c, z_c) \text{ and the focus of a parabola} \\
d_{df} &= \text{distance between the directrix and the focus of a parabola} \\
m_i &= \text{slope of the incident ray} \\
m_n &= \text{slope of the normal at a point } (r_p, z_p) \\
m_r &= \text{slope of the reflected ray} \\
m_t &= \text{slope of the tangent at a point } (r_p, z_p) \\
R &= \text{radius of the spherical telescope} \\
T_c &= \text{critical temperature for flux pinning} \\
r &= \text{polar coordinate} \\
r_c &= \text{r coordinate of the center of a mirror segment} \\
r_m &= \text{r-intercept of the normal} \\
r_{ir} &= \text{r-intercept of the reflected ray} \\
r_p &= \text{r coordinate of a point on a parabola} \\
r_v &= \text{r coordinate of the vertex of a parabola} \\
z_c &= \text{z coordinate of the center of a mirror segment} \\
z_p &= \text{z coordinate of a point on a parabola} \\
z_v &= \text{z coordinate of the vertex of a parabola}
\end{align*}

1. Introduction

As astronomers focus on the direct detection of exoplanets, the study of ever more distant objects, and more detailed Earth observation, there is a need for space telescopes that are far more powerful than those currently in use. The power of a space telescope depends upon its angular resolution and its light-gathering ability. The

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angular resolution of a telescope represents the smallest angular separation at which two objects can be resolved; the smaller the angular resolution, the closer together the two objects can be. According to Rayleigh’s criterion, the angular resolution $\theta$ is limited by both the wavelength $\lambda$ of the light being detected and the diameter $d$ of the aperture:

$$\sin \theta = 1.22 \frac{\lambda}{d} \quad (1.1)$$

As a result, decreasing the wavelength or increasing the size of the aperture improves the angular resolution. The light-gathering ability of a telescope determines how bright an object must be before it can be detected; the more photons the telescope collects, the fainter the objects it can detect. The light-gathering ability can be improved by using more sensitive detectors, which require fewer photons to detect an object, or by using a larger aperture, which can collect more photons. Alternatively, it can be improved by increasing the integration (or exposure) time, but the application of the telescope might limit the amount of time that is possible; images of Earth-based targets cannot be integrated indefinitely because of diurnal changes and any relative motion between the Earth and the telescope. The telescope design outlined in this paper seeks to improve both the light-gathering ability and the angular resolution by using a larger aperture.

Since the size of a telescope scales roughly with the size of its aperture, constructing a more powerful telescope by increasing the aperture means constructing a larger telescope. Unfortunately, the limited capacity of a launch vehicle places an upper bound on the size of monolithic telescopes that can be assembled on the ground and sent into orbit. To maximize the size of a monolithic telescope that can be launched, several ingenious strategies have been developed. These strategies include designing inflatable structures and using creative folding techniques to minimize the volume of the telescope. The 6.6m primary mirror of the James Webb Space Telescope, for example, is designed to fold compactly to allow the telescope to fit into a 5m shroud. However, although creative packaging strategies allow larger pre-assembled telescopes to be launched, there is still an upper limit on their size; beyond a certain size, deployable telescopes cannot fit in any current launch-vehicle fairing.

To create even larger space telescopes, an alternative approach is to launch several components that are either mechanically assembled in space or designed to fly in formation. The Eyeglass is an example of a system that adopts this approach. Although an intricate origami-like folding technique allows its 25m lens to fit inside its launch vehicle, the telescope has a focal length of approximately 1 km, necessitating the use of two spacecraft: one for the lens and one for the detector. These spacecraft are intended to fly in formation since tethers might transmit undesirable vibrations. Another example of a space telescope that relies on formation flight is the Terrestrial Planet Finder Interferometer (TPF-I). In order to complete its scientific objectives, TPF-I requires an angular resolution of <50 mas, which translates into a primary mirror diameter of over 40m in a monolithic telescope. Since the challenge of packaging such a large mirror is not insignificant, the telescope has an interferometric design instead, with the components designed to fly in formation because a sufficiently large boom could not be designed to fit inside the Ariane 5 ECA launch vehicle. Although formation flight enables the construction of larger space telescopes, it is not a perfect solution: active control is required to maintain the relative position of each component and to prevent collisions or misalignment. Ideally, the design of a large space telescope would neither depend upon the size of the launch vehicle nor require active control; such a design could be scaled arbitrarily large, and the resulting telescope would be passively stable.

The phenomenon of magnetic flux pinning might provide a way to connect the telescope components that overcomes the limitations of formation flight or a mechanical support structure. A type of interaction between a magnet and a type II superconductor, flux pinning is analogous to a damped spring force that acts over a distance. A simple model is that of a magnet and a superconductor connected by a virtual spring and damper. This interaction is passively stable and requires only the power needed to keep the superconductor cooled below a critical temperature. Section III presents an architecture for a very-large-aperture flux-pinned space telescope.

II. Flux Pinning

According to the frozen-image model, two image magnets form within a superconductor that is cooled below its critical temperature $T_c$ (88K for superconductors made of yttrium barium copper oxide) in the presence of a permanent magnet. One is a mirror image of the permanent magnet, located the same distance from the superconductor boundary as the permanent magnet but with an opposite dipole vector. This image is free to move within the superconductor as the permanent magnet moves, and the interaction between the permanent magnet and this image gives rise to Meissner repulsion. The second image that forms is located the same distance from the
superconductor boundary as the permanent magnet, and it has the same dipole vector. Unlike the first image, this second image is frozen in place; it does not move with the permanent magnet. The interaction between the permanent magnet and the frozen image gives rise to flux pinning (Figure 1).

Flux pinning offers three important features for space-system design. First and foremost, it is a passively stable interaction that fixes the position and orientation of the superconductor relative to the permanent magnet. Current research suggests that flux pinning can act over distances on the order of tens of centimeters and fix the relative position of the magnet and the superconductor with nanometer precision without any active control. Second, flux pinning can be used to create a permanent equilibrium configuration. Since the frozen image is fixed in place, it is unaffected by the position and orientation of the permanent magnet, allowing the system to remember its initial configuration. As a result, even if the magnet and superconductor are separated by more than the maximum distance over which flux pinning can act, they move towards their initial configuration if they are brought within flux-pinning distance again. Finally, since flux pinning is possible only when the superconductor temperature is below $T_c$, it can be turned on or off simply by changing the temperature of the superconductor. If a magnet and a superconductor are flux pinned in one configuration, their relative position and orientation can be adjusted by heating the superconductor above $T_c$, relocating and reorienting the magnet and superconductor, and cooling the superconductor below $T_c$. These three characteristics allow flux pinning to create passively stable, non-contacting, reconfigurable interfaces. The construction of and uses for these interfaces is a topic currently under investigation.

### III. The Telescope Design

The non-contacting, reconfigurable interfaces made possible by flux pinning have a potential application in the construction of a large space telescope. Traditionally, segmented-mirror telescopes rely on a stiff mechanical frame to fix the position and orientation of each mirror segment, which poses a problem for space-bound telescopes. Limited by the dimensions of a launch-vehicle fairing, large space telescopes must either fold up or be launched in sections and assembled in space. For telescopes relying on a mechanical frame, neither is a trivial problem. For a telescope

![Figure 1. The Frozen Image Model](image)

The picture on the left shows the formation of the mobile and frozen images as the superconductor is cooled below its critical temperature in the presence of a permanent magnet. The picture on the right shows the mobile and frozen images at a later time, after the permanent magnet has been moved.

![Figure 2. A possible module architecture](image)
whose mirror segments are held in place by flux pinning, however, space assembly might be less of a concern. Since flux-pinning modules can be made to find one another and latch on without active control, traditional proximity operations or extravehicular activity might be unnecessary\textsuperscript{10}. Additionally, since flux-pinning interfaces are reconfigurable, mirrors could be added or replaced without maneuvering the entire assembly. As a result, the use of flux-pinning modules could simplify the assembly of large space telescopes significantly, while also allowing for non-intrusive future repairs and modifications (Figure 2).

Here we consider a sparse-aperture space telescope design in which a detector lies at the center of a spherical shell of identical deformable mirror segments mounted on flux-pinning modules. Initially, the detector and some

![Figure 3a](image1.jpg) ![Figure 3b](image2.jpg) ![Figure 3c](image3.jpg)

Figure 3. The Telescope and a Concept of Operations for Assembly

3a) The detector (blue) and the flux-pinning mirror modules (gray) are released from the launch vehicle. 3b) The mirror modules use CDGPS measurements of their distances from the detector to navigate to a distance $R$. At the same time, a simple swarming law directs the modules to flux pin in a web-like pattern. 3c) The assembled telescope.  
Earth image courtesy of NASA

number of mirror modules are launched together and separated from the launch vehicle after it reaches the desired orbit. Then, using a carrier-phase differential GPS-based attitude determination system\textsuperscript{11}, the mirror modules measure their displacement from the detector. Using these measurements in a closed-loop control scheme, the mirror modules autonomously navigate to a distance $R$, placing the mirror segments with their centers tangent to a spherical shell and their reflecting surfaces pointing inward. The flux-pinning modules control the position and orientation of each mirror. To ensure that the modules are distributed over the sphere while remaining close enough to flux pin, a simple swarming command directs them to form a web-like pattern, with strands of closely spaced modules sprawling along the surface of the sphere; the empty space between the strands allows light into the telescope (Figure 3).

IV. Mirror Deformation

Unfortunately, although spherical mirrors are relatively easy to construct, they do not focus incident light to a single point; as a result, images formed by spherical mirrors suffer from spherical aberrations. To correct for this problem without destroying the simple overall architecture of the telescope, each mirror segment capable of reflecting the incident light deforms so that its reflecting surface is shaped like a segment of the paraboloid passing through the center of the mirror with its focus at the detector at the center of the sphere. (Since the mirror segments are distributed over the surface of a sphere, some of them will lie in the path of the incident light. These mirror segments are incapable of reflecting the incident light to the detector, but they block only a small fraction of the light.) The resulting mirror arrangement combines the focusing

![Figure 4](image4.jpg)

Figure 4. The circular shell
power of a conventional paraboloidal mirror with the ease and versatility of maintaining a spherical formation, and it

gives the telescope the additional advantage of being able to change its field of view relatively quickly; the time

required to switch its field of view is limited only by the time it takes to rotate the detector and deform the mirrors

appropriately. An alternative configuration in which an optical element corrects the spherical aberration requires the

correcting element to fly between the central detector and the spherical shell. The mass center of this element is

offset from those of the shell and the detector; so, stationkeeping these various subsystems is nontrivial and requires

constant actuator input. In contrast, the paraboloidal mirrors allow the mass center of the detector to follow that of

the sphere, leaving only environmental perturbations to be corrected.

To verify that this mirror arrangement reflects the incident light to the center of the sphere, consider first the

two-dimensional case of linear mirror segments placed with their centers tangent to a circle and deformed into

parabolic segments with a common focus at the center of the circle (Figure 4). The three-dimensional case will be

obtained by rotating the two-dimensional case about the axis of symmetry, the optic axis. The circle has a radius $R$

centered on the origin of the rz-plane. For simplicity, the optic axis is along the z-axis, and the source of the incident

light is located at $z = -\infty$. For any choice of $r$ such that $-R \leq r \leq R$, there are two points $(r,z)$ that lie along the

circle, where

$$z = \pm \sqrt{R^2 - r^2}$$

(4.1)

However, since the incident light is traveling in the positive z-direction, only mirrors placed at points with positive $z$

values can reflect it to the detector at the origin. Mirrors placed at points with negative $z$ values block the light from

entering the telescope, and mirrors for which $z = 0$ are oriented parallel to the incident light. Since only the mirrors

that can reflect the incident light are of interest, only mirrors placed at points with positive $z$ values need to be

considered. Additionally, in order to show that any mirror that reflects the incident light will reflect it to the center

of the shell, questions about whether or not a mirror is blocked will be ignored.

For a mirror segment whose center lies at $(r_e, z_e)$, where $r_e$ and $z_e$ meet the conditions described above, there is

only one parabola passing through $(r_e, z_e)$ whose axis of symmetry is the z-axis and whose focus is the center of the

shell. (The z-axis is chosen as the axis of symmetry since a parabola reflects incident light to its focus only when

that light is parallel to its axis of symmetry.) The equation for this particular parabola can be derived using simple

graphology.

In general, a parabola symmetric about the z-axis and opening downward is described by the equation

$$\left( r - r_v \right)^2 = -4a \left( z - z_v \right)$$

(4.2)

where $r_v$ is the $r$-coordinate of the vertex, $z_v$ is the $z$-coordinate of the vertex, and $a$ is the distance between the focus

and the vertex. Additionally, for any point on the parabola, the distance between that point and the focus is equal to

the distance between that point and the directrix. So, since the distance from $(r_e, z_e)$ to the focus is

$$d_f = \sqrt{\left( r_e - 0 \right)^2 + \left( z_e - 0 \right)^2} = R$$

(4.3)

the distance from $(r_e, z_e)$ to the directrix must also be $R$. Since the parabola opens downward and is symmetric

about the z-axis, the directrix must be a line parallel to the r-axis. As a result, the directrix must be the line

$$z = z_v + R$$

(4.4)

Since the distance between the directrix and the focus is

$$d_d = z_v + R = 2a$$

(4.5)

the distance between the vertex and the focus is

$$a = \frac{z_v + R}{2}$$

(4.6)
and the vertex is the point

\[ (r_z, z_v) = \left(0, \frac{z_v + R}{2}\right) \]  

(4.7)

By substituting Equation (4.6) and Equation (4.7) into Equation (4.2), we find that the equation of the desired parabola is

\[ z = -\frac{1}{2(z_v + R)} r^2 + \frac{z_v + R}{2} \]  

(4.8)

Finally, Equation (4.8) can be generalized to the three-dimensional case by rotating about the axis of symmetry, the \( z \)-axis. Mathematically, this means making the substitutions

\[ x = r \cos \theta \]  

(4.9)

and

\[ y = r \sin \theta \]  

(4.10)

where \( \theta \) is the angle between \( r \) and the positive \( x \)-axis. Making these substitutions, we find that the three-dimensional mirror deformation equation is

\[ z = -\frac{1}{2(z_v + R)} (x^2 + y^2) + \frac{z_v + R}{2} \]  

(4.11)

If a mirror segment placed along the surface of the sphere is deformed according to Equation (4.11), then the entire surface of the mirror reflects the incident light to the detector at the center of the sphere. Numerically, this can be shown by plotting the reflected ray for an arbitrary point \((r_p, z_p)\) on the mirror segment and checking whether the rays pass through the center of the sphere. The equation representing this ray can be found by using a combination of geometry and Snell’s Law.

For simplicity, consider first the two-dimensional case of linear mirror segments placed along a circular shell. On a diagram of the incident ray, the reflected ray, and the line tangent to the mirror at \((r_p, z_p)\), there are two right triangles of interest, as shown in Figure 5. One of these triangles is the triangle formed by the normal, the \( r \)-axis, and the line \( r = r_p \). The vertices of this triangle are easily determined, and trigonometry can be used to relate the lengths of its sides to the angle between the incident ray and the line tangent to the mirror segment. The other triangle of interest is the one formed by the reflected ray, the \( r \)-axis, and the line \( r = r_p \). The lengths of the legs of this triangle determine the slope of the reflected ray, and although these lengths are not all immediately apparent, they can be related to the angle between the reflected ray and the line tangent to the mirror segment. Snell’s Law states that the angle between the incident ray and the tangent must be the same as the angle between the reflected ray and the tangent.
Therefore, it is possible to relate the slope of the reflected ray to the easily determined lengths of the legs of the first triangle.

At an arbitrary point \((r_p, z_p)\), the mirror segment has the slope

\[
m_t = \frac{d}{dr} \left( \frac{-1}{2(z_c + R)} r^2 + \frac{z_c + R}{2} \right) = \frac{-r_p}{z_c + R}
\]

so the slope of the normal is

\[
m_n = \frac{-1}{m_t} = \frac{z_c + R}{r_p}
\]

When extended to the \(r\)-axis, the normal has the \(r\)-intercept

\[
r_{in} = \frac{-z_p}{m_n} + r_p
\]

So, from basic trigonometry, the angle between the incident ray and the tangent line is given by

\[
\theta = \frac{\pi}{2} - \tan^{-1} \left( \frac{r_p - r_{in}}{z_p} \right)
\]

Since Snell’s Law states that the angle at which the incident ray strikes the mirror must be the same as the angle at which it reflects, it is possible to construct another right triangle and use basic geometry and Equation (4.15) to determine the slope of the reflected light. Since the reflected ray has the \(r\)-intercept

\[
r_r = r_p - z_p \tan \left[ 2 \tan^{-1} \left( \frac{r_p - r_{in}}{z_p} \right) \right]
\]

the slope of the reflected ray is

\[
m_r = \frac{z_p}{r_p - r_r} = \frac{1}{\tan \left[ 2 \tan^{-1} \left( \frac{r_p - r_{in}}{z_p} \right) \right]}
\]

Inserting Equation (4.17) into the point-slope equation, we find that the reflected ray is given by

\[
z = \frac{1}{\tan \left[ 2 \tan^{-1} \left( \frac{r_p - r_{in}}{z_p} \right) \right]} \left( r - r_p \right) + z_p
\]

As a Figure 6 reveals, the reflected ray does pass through the center of the shell. Since this statement is true for an arbitrary point on an arbitrary reflecting mirror, it is true for any point on any reflecting mirror. So, the mirror arrangement described above does focus the incident light to a single point, the center of the circular shell. In addition, since the three-dimensional case is simply a rotation of the two-dimensional case about the axis of symmetry, the three-dimensional case described above also focuses the incident light to a single point, the center of
the spherical shell (Figure 7). However, this assertion can be verified explicitly by substituting Equation (4.9) and Equation (4.10) into Equation (4.18).

**Figure 6.** Two-dimensional ray diagram for flat mirror segments placed on a circular shell and reshaped into parabolic segments with a common focus

The black line corresponds to the circular shell, the blue lines correspond to reflected rays, and the red lines correspond to individual mirror segments. The inset on the top right provides a close-up view of one mirror segment and its relation to the circular shell.
V. Swarming Behavior

For a telescope whose detector lies at the center of a spherical shell of flux-pinned mirror segments, spaces between the mirror segments allow light to enter the telescope. To maintain the overall isotropy of the telescope, the mirrors ideally would be distributed evenly over the sphere, rather than clumped in some locations and sparse in others. Full coverage of the UV-plane might be achieved as these mirrors move, whether by a rolling motion or through some stochastic behavior. However, the limited range of flux pinning restricts the distances between the mirrors. As the mirrors spread out over the sphere, they must remain close enough to flux pin; otherwise, no force would maintain the formation, and the modules would drift apart. One way to ensure that the mirrors remain sufficiently close while maintaining statistical coverage of the sphere is to measure the position of each mirror and use a complex control law to direct each mirror to some predefined location on the sphere. However, a much simpler approach is to use a swarming command to control the assembly of the mirrors.

A swarming command is a simple, local rule that each member of a large collection, or swarm, follows when interacting with its neighbors. This rule is the same for every member of the swarm, and it depends upon only

Figure 7. Three-dimensional ray diagram for flat mirror segments placed with their centers tangent to a sphere and reshaped into paraboloidal segments with a common focus

Figure 8. A natural swarm
Photo courtesy of Vicki Barefoot-Gersh
relationship between the member and its neighbors; it does not rely on any global information, like the exact position within the swarm. However, the key feature of a swarm is that even though the individual members follow a simple, local rule, the swarm as a whole exhibits emergent behavior, an orderly behavior of the group. Swarming commands are used in nature, for example, to control the behavior of flocks of birds or schools of fish. Within a flock of birds, each individual bird follows a simple rule that offsets it slightly from the bird in front, and the end result is that the entire flock flies in a V-formation, as shown in Figure 8. In this case, the swarming command is the rule that each individual bird follows, and the emergent behavior is the V-formation. Similarly, in a school of fish, each individual fish follows a rule that tells it to stay close to its neighbors and move as they do, and the emergent behavior is a shimmering wall of fish that seems to move as one seamless unit.

With regard to the telescope, the swarming command is a simple physical law that governs the interactions between one mirror module and its neighbors, and the emergent behavior is the overall distribution, strands of closely spaced modules that sprawl along the sphere, for example (Figure 9). A likely solution involves placing three flux-pinning attachment points separated by 90° on each mirror module. Then, even though every module has three attachment points, only some small percentage of them can use all three; the majority are allowed to form attachments at only the two points separated by 180°. Preliminary investigations into this type of swarming command suggest that the emergent behavior depends upon the number and placement of the attachment points and the relative magnitudes of the equilibrium separation distance, the size of the module, the distance over which flux pinning can occur, and the strength of the flux-pinning interaction.

VI. Conclusion

There are several potential advantages to the telescope design proposed here. First and foremost, the design is scalable. Although the initial radius of the telescope is determined by the number of mirror segments that are launched, the radius can grow if additional mirror segments are added later. As a result, the aperture of the telescope can be increased gradually, spreading the cost over time. The reconfigurable nature of the flux-pinning interfaces offers another advantage: unlike the Hubble Space Telescope, which required an expensive manned mission to repair a defective mirror, a telescope whose mirror segments were flux-pinned could be repaired autonomously. Misplaced or imperfectly deformed mirror segments can be repositioned or reoriented remotely, and since the mirror segments are interchangeable, removing and replacing any damaged or destroyed segments can require minimal human involvement. In addition, since flux pinning requires no active control, the telescope is passively stable in the event of a software-related failure and able to maintain its shape using only the minimal amount of power required to cool the superconductors. One final advantage of this design stems from its overall geometry. As a spherical shell of mirror segments with a detector floating at the center, the telescope is roughly isotropic, so it can be repointed by rotating the central detector and deforming the mirror segments appropriately. If the mirrors are capable of deforming sufficiently quickly, then this telescope could repoint in less time than an equivalently sized telescope requires to slew, lending the telescope high agility.
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References