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Stochastic Space Exploration with Microscale Spacecraft

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Advancements in microelectronics have enabled the construction of low-cost "ChipSats" – entire spacecraft integrated onto a few square centimeters of silicon. This paper explores the use of statistical methods to plan exploration missions using large collections or "clouds" of such spacecraft. The equations governing the time-evolution of a cloud of ChipSats are developed in terms of probability distributions and several methods for solving them are discussed. The passive evolution of ChipSat clouds are then explored under the influence of a spherical gravitational potential and a case study is performed for an asteroid impact mission.

Nomenclature

f	=	probability distribution function
H	=	Hamiltonian
i	=	coordinate index
j,k	=	moment indices
M	=	moment of a probability distribution
m	=	total mass
p	=	momentum coordinate
q	=	position coordinate
r,θ,ϕ	=	spherical coordinates
T	=	kinetic energy
t	=	time
V	=	potential energy
Φ	=	gravitational potential
μ	=	standard gravitational parameter

I. Introduction

RAPID advancements in the integration and miniaturization of consumer electronics have made a huge variety of low-cost off-the-shelf components available for possible use in spacecraft, from microprocessors to GPS receivers. Previous investigations have explored the prospect of using semiconductor fabrication techniques to mass-produce chip-scale satellites[1] ("ChipSats"), as well as the unique dynamics of very small spacecraft[2]. The low cost and small mass of such systems introduce the realistic possibility of simultaneously deploying and operating thousands of spacecraft, enabling exciting new possibilities for distributed sensing.

ChipSats could enable entirely new categories of science missions such as large-scale in-situ surveys of planetary atmospheres using MEMS or nanofluidic sensors[4]. Earth's mesosphere, for example, could be studied in ways that are impractical or impossible with current sounding rocket experiments[5]. A large ensemble of ChipSats with several different chemical detectors could also be used as a heterogeneous swarm of tiny impactors to determine the surface composition of an asteroid. ChipSats would uniquely allow thousands of data points to be collected over a large spatial volume.

These missions, like all space missions, are fraught with uncertainty. Spacecraft designers and mission planners have traditionally countered uncertainty with redundancy: by using components that have been tested and qualified in conditions much more severe than those expected, by incorporating backup subsystems into designs, by designing control systems with extra margin, by budgeting for extra fuel, etc. While these approaches certainly lead to a more reliable spacecraft, they also lead to a larger, heavier, more expensive spacecraft that takes longer to build. The

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same approach simply cannot be used at the scale of the ChipSat, where the benefits of small scale and low cost would quickly be eroded.

The ability to deploy thousands of ChipSats for less than the cost of one large satellite enables an entirely new approach to handling uncertainty in a space system. If, instead of trying to get each individual ChipSat to meet some mission objective, we assign an objective or set of objectives collectively to a cloud of thousands of ChipSats, we've instantly gained far more robustness than could ever be achieved with a single monolithic spacecraft. In addition to its inherent robustness to individual failures, the ChipSat cloud allows missions to be planned in a statistically rigorous way, such that all uncertainties can be fully taken into account and quantitative answers to relevant mission design questions can be computed with confidence intervals.

II. ChipSat Hardware

The Sprite is a functional proof of concept for a chip-scale spacecraft developed at Cornell[3] that has the ability to harvest solar energy and transmit data to ground stations from low Earth orbit. Current prototypes are 3.8 centimeters by 3.8 centimeters and are constructed entirely out of commercial off-the-shelf components on standard printed circuit boards. Three such devices were carried to the International Space Station on board STS-134 as part of the MISSE-8 experimental pallet, allowing real-world tests of communication systems and the survivability of COTS electronics in the LEO environment.

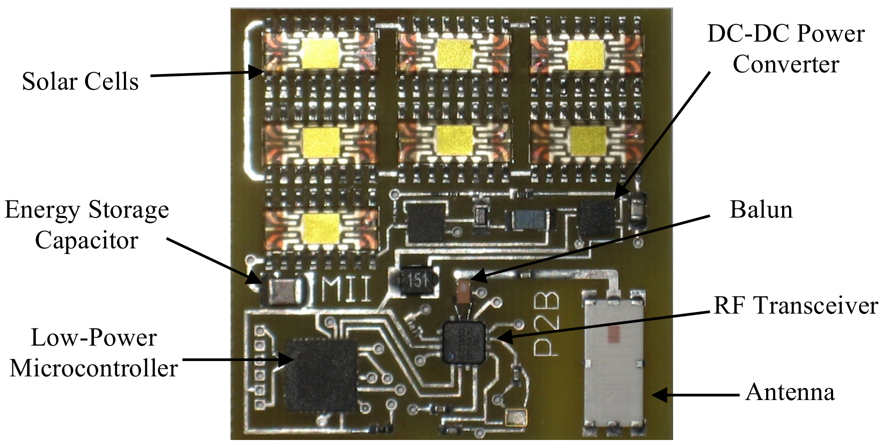

Figure 1: Sprite Spacecraft.

While currently an order of magnitude larger than the eventual goal system, the majority of the Sprite's mass and area are due to the plastic packaging on each integrated circuit. By combining all of the die from these ICs onto a single substrate in what is known as a multi-chip module[6], the device can be shrunk considerably. Initial investigations into packaging techniques were performed at Sandia National Laboratories' Center for Integrated Nanotechnology in 2009, and work in this area is currently progressing in collaboration with Draper Laboratories. Future versions of the Sprite will include energy storage capability in the form of a thin-film battery, allowing operation during eclipse, as well as a GPS receiver.

The ultimate goal of the Sprite project is to produce an application-specific integrated circuit (ASIC), a device fabricated on a single silicon wafer using semiconductor fabrication techniques. Such a ChipSat would require no special integration or handling procedures, occupy about 1 cm of total area, and be a few tens of microns thick. It would incorporate all the electronics necessary to perform meaningful sensing and transmit results, combining, for example, a lab-on-chip[4], solar cells, a microcontroller core, and a low-power RF transmitter. It is possible to build such a device using current technology, and if produced in large numbers, economies of scale could bring the per-chip price down to a few dollars.

III. ChipSat Cloud Dynamics

While the motion a single ChipSat can be well understood using familiar techniques from dynamics[2], the prospect of deploying tens or hundreds of thousands presents some additional challenges. Taking the viewpoint that what matters is the overall statistical picture of the cloud, rather than the motion of each individual spacecraft, we can decouple the computational effort required from the number of ChipSats.

A. The Collisionless Boltzmann Equation

We begin by assuming that there are sufficiently many ChipSats such that it makes sense to describe the entire ensemble by defining a probability density function on the six-dimensional phase space $f(q_i, \dot{q}_i, t)$. To find the differential equation governing the evolution of this distribution, we begin by expanding its time derivative with the chain rule.

$$\frac{\mathrm{d}f}{\mathrm{d}t} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial q_i}\frac{\mathrm{d}q_i}{\mathrm{d}t} + \frac{\partial f}{\partial \dot{q}_i}\frac{\mathrm{d}\dot{q}_i}{\mathrm{d}t} \tag{1}$$

From here, we make the assumption that the dynamics of each ChipSat are governed by Hamilton's Equations, allowing us to take advantage of Liouville's Theorem[8], which states that phase space density is conserved along solution trajectories. Put another way, if one thinks of the flow of the differential equations in phase space as analogous to the flow of a fluid, the flow is incompressible. This means that the total derivative, or, keeping with the fluid analogy, the "convective derivative" in Eq. (1), is zero. The resulting partial differential equation, Eq. (2), is known as the collisionless Boltzmann Equation[7,9].

$$\frac{\partial f}{\partial t} + \frac{\partial f}{\partial q_i}\dot{q}_i + \frac{\partial f}{\partial \dot{q}_i}\ddot{q}_i = 0 \tag{2}$$

To illustrate how one might apply Eq. (2) to the case of a ChipSat cloud orbiting a body with a spherically symmetric gravitational field, we can use spherical coordinates and substitute in the familiar inverse-square law for $\ddot{q}_i$, giving Eq. (3).

$$\frac{\partial f}{\partial t} - \frac{\mu}{r^2}\frac{\partial f}{\partial \dot{r}} + \dot{r}\frac{\partial f}{\partial r} + \dot{\theta}\frac{\partial f}{\partial \theta} + \dot{\phi}\frac{\partial f}{\partial \phi} = 0 \tag{3}$$

It is worth discussing the range of applicability and limitations of the collisionless Boltzmann Equation. First, as the name implies, it does not account for collisions between ChipSats. This should not present a problem for our application, as collisions will be very rare unless the density is extremely high in a given region. Second, since we assumed that the dynamics of the ChipSats are Hamiltonian, all forces on them must be conservative, arising from a potential. This means that gravitational and electromagnetic forces, as well as control inputs defined using artificial potential functions, can be accounted for, but dissipative mechanisms, such as atmospheric drag, cannot. As we will see in the next section, however, this can be worked around with numerical solution methods.

B. Solution Methods

Eq. (2) is, in general, a nonlinear partial differential equation. Analytic solutions to such equations can very rarely be found, and so approximate and numerical techniques must be used. The most common of these techniques in engineering applications are probably grid-based methods like finite element. Unfortunately, such methods are very poorly suited to the collisionless Boltzmann Equation because the number of grid nodes needed to discretize a given volume increases exponentially with the number of dimensions. To keep track of three degrees of freedom, as in Eq. (3), we need a six dimensional phase space, and if we wanted to keep track of attitude as well, the problem gets many orders of magnitude worse, becoming intractable.

Another common technique, the Monte Carlo method[10], is much better suited to higher-dimensional problems. The idea is to sample the initial distribution $f(q_i, \dot{q}_i, t_0)$ and propagate the trajectories of each sample point through the ODEs governing the dynamics of an individual ChipSat. Various methods are then available for constructing an approximation of the distribution $f(q_i, \dot{q}_i, t_f)$ from the set of propagated sample points. The Monte Carlo method is simple and robust, and since it makes use of the ODEs for the individual ChipSats and does not rely on the Hamiltonian assumption of the collisionless Boltzmann Equation, it can handle non-conservative forces like drag. It can, however, require a significant computational effort for situations where high accuracy is required, as many sample points must be used.

A third option is to expand the distribution function in terms of its moments[7,9]. ODEs for the moments can then be generated and solved via standard methods such as Runge-Kutta. Re-writing the distribution function in terms of position and momentum, rather than velocity, we have $f(q_i, p_i, t)$. For clarity, the subscripts on q_i and p_i will be omitted for the rest of this section, but note that they are still implied. The moments of $f(q, p, t)$ are defined by Eq. (4), where $\langle\cdot\rangle$ denotes the expectation value.

$$M_{j,k} = \langle q^j p^k \rangle = \int q^j p^k f(q,p,t)\, dq\, dp \tag{4}$$

We now define the Hamiltonian[7] for the entire ChipSat cloud in Eq. (5), where $\Phi(q,t)$ is the gravitational potential.

$$H = T + V = \frac{1}{2m} \iint p^2 f(q,p,t)\, dq\, dp + m \iint \Phi(q,t) f(q,p,t)\, dq\, dp \tag{5}$$

Note that the kinetic energy is simply $\frac{1}{2m} M_{0,2}$ and also that by expanding $\Phi(q,t)$ in a Taylor series, we can write the potential energy in terms of the $M_{j,k}$. By doing this, we can re-write the Hamiltonian entirely in terms of the moments of the distribution, truncating to whatever order is necessary to achieve the desired accuracy. Finally, we get ODEs for the time-evolution of the moments by taking their Poisson bracket[7] with the Hamiltonian.

$$\frac{dM_{j,k}}{dt} = \{M_{j,k}, H\} = \frac{\partial M_{j,k}}{\partial q}\frac{\partial H}{\partial p} - \frac{\partial M_{j,k}}{\partial p}\frac{\partial H}{\partial q} \tag{6}$$

The moment approach has several advantages – it allows one to easily trade computational effort for accuracy, and is the least computationally intensive of the methods discussed, possibly making it suitable for real-time use in control and estimation algorithm on the severely constrained hardware of the ChipSat. One disadvantage is that the derivation of the moment ODEs relies on the same Hamiltonian assumptions as the collisionless Boltzmann Equation, making them unsuitable for use in situations where drag has a significant effect.

IV. Asteroid Impact Mission

To give a concrete illustration of the use of statistical methods as described in the previous sections, we will analyze a hypothetical mission to impact a Near-Earth Object, 2011 MD, which passed within 12,000 km of the Earth's surface on June 27th 2011[11]. The basic outline for the mission consists of deploying a large number of ChipSats in a circular Earth orbit in such a way that the maximum number impact 2011 MD at its perigee. We will calculate how many ChipSats will need to deploy to be 90% confident that at least one will survive impact and transmit sensor data from the asteroid's surface.

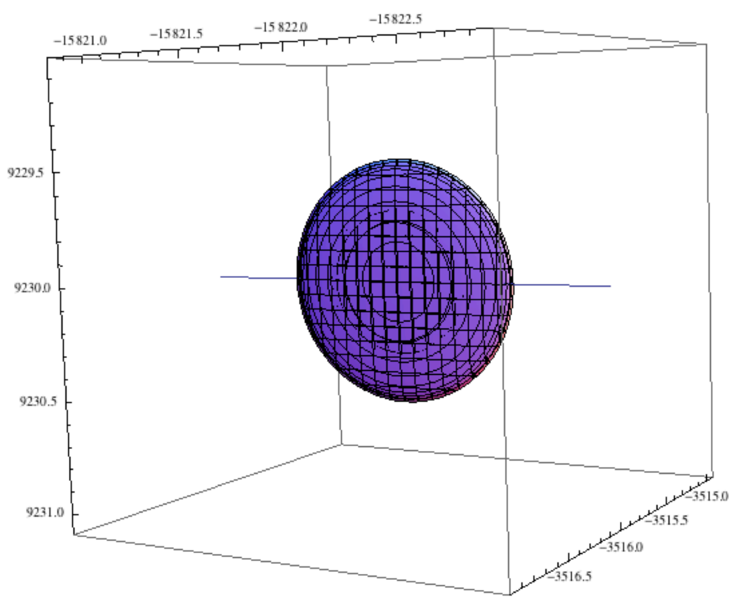

Figure 2. Contour plot of ChipSat position distribution along nominal asteroid trajectory at impact time.

Like most other NEOs, relatively little is known about 2011 MD. Only very rough estimates of its size[11] and guesses at its composition[11] are available. It seems the only concrete data available is in the form of NASA-generated ephemeris tables[12] giving position and error estimates. Ephemeris data was used to generate a probability density function for the location of the asteroid. To maximize the probability of impact, the ChipSat distribution was chosen to match the asteroid uncertainty distribution at perigee. The better the position knowledge of the asteroid, the tighter the ChipSat distribution can be, and the fewer ChipSats will be required. For this particular case, standard deviation values were approximately 23 meters in the radial direction and 334 meters in the plane perpendicular to the radius vector.

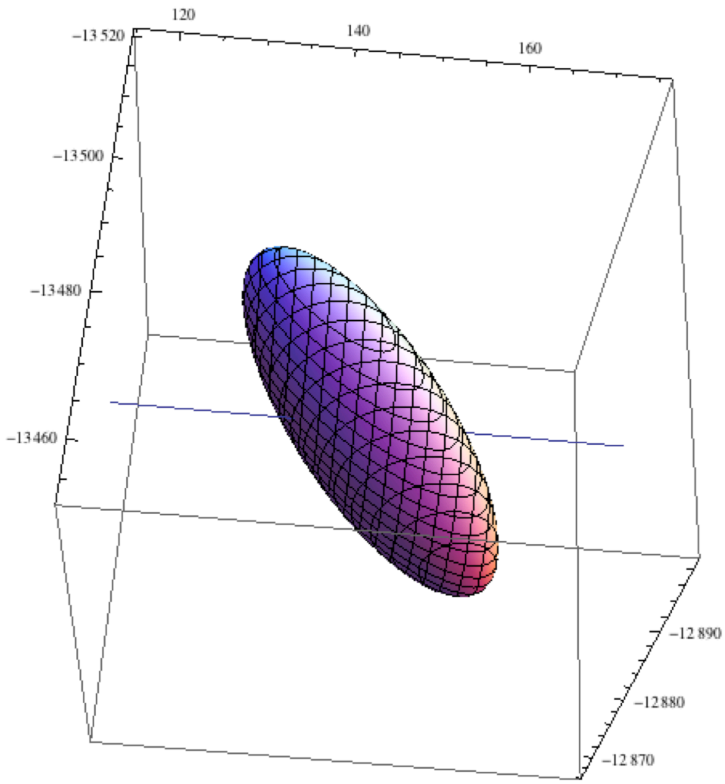

Figure 3. Contour plot of ChipSat position distribution along nominal circular orbit two hours before impact.

Knowing how the ChipSat cloud should be distributed at the time of impact naturally leads to the question of how the cloud should be distributed when it is deployed at some earlier time. To determine this, a Monte Carlo simulation was performed in which 1000 sample points were taken from the distribution at the time of impact and their trajectories were propagated backward. As an example, assuming the cloud was to be deployed 2 hours before 2011 MD reached perigee, it would be shaped as in Fig. 3. Note that, as one would expect, ChipSats closer to the Earth lag those farther out to compensate for differences in orbital velocity, so that the distribution is shaped like Fig. 2 when the point of impact is reached.

The second piece of information necessary to compute impact probabilities is the size of the asteroid. The only information available puts its diameter "between 10 and 50 meters"[11]. From this, we assume the error to be normally distributed with 50 and 10 meters being three standard deviations above and below the mean respectively. Impact probabilities can then be computed by integrating the total probability flux through a circle of the appropriate diameter under the action of the orbital dynamics. Taking into account uncertainty in both asteroid trajectory and size, the expected probability of impact is approximately 1.4%, with a 90% confidence interval of .16% to 2.5%. Thus, to be 90% confident that at least one ChipSat will impact the asteroid, at least 625 should be deployed, and given a deployment of that number, 8.75 would be expected to impact.

Impacting the asteroid does not, of course, imply survival. On the contrary, one would probably expect the majority of the ChipSats to be destroyed upon impact given a relative impact velocity of about 2 km/s. Lacking any empirical data, a basic first-principles model was used to gain a very rough estimate of the survival statistics. Three impact failure modes were initially considered: bending, compression, and thermal, while the ChipSat was modeled as a square of silicon measuring 1.7 cm on a side and 100 μm thick. It became clear that only the bending mode would lead to fracturing of the chip, with a dependence on the angle of incidence.

Traveling at 2 km/s, a ChipSat has to shed about 140 joules of excess kinetic energy to come to rest on 2011 MD. Most Asteroids have a regolith a few centimeters deep,[13] which can effectively absorb kinetic energy stored in the velocity component parallel to the surface, but not the velocity component perpendicular to the surface. This means that the energy put into bending of the ChipSat is proportional to $\sin^2 \theta$, where θ is measured with zero being parallel to the surface. Given that fracture strength of silicon varies from 3 to 5 GPa with 90% confidence[14], this tells us that the bending mode of a ChipSat can absorb between .049 and .13 joules without fracturing. Assuming a uniform distribution of impact incidence angles, between 1% and 2% of the ChipSats that impact the asteroid will not fracture. Combining this with our previous impact percentages gives a successful impact probability of .021% with a 90% confidence interval of .0016% to .05%. To be 90% confident that at least one ChipSat will impact the asteroid and survive, at least 62,500 should be deployed, with an expectation of 13.125 survivors. While this number might seem very high, keep in mind that many ChipSats could be packed into a box measuring less than 20 cm on a side, and future large-scale production could reduce costs to a few dollars per ChipSat.

V. Conclusion

The ability to model and fully account for uncertainty in a space mission leads to increased capability, higher success rates, and lower costs. ChipSats are unique in the degree to which they enable the application of statistical modeling to mission design. Simply because there are many of them, the designer is dealing with a large sample size, and statistical inferences can be made with greater confidence.

ChipSats are a promising new technology for space exploration, offering not only a new size scale for spacecraft, but also a new way of thinking about their design and mission planning. The potential to use vast numbers of them for sensing could open up new opportunities in fields like atmospheric sensing and planetary science. ChipSats could also be used in conjunction with more conventional spacecraft to add new mission capabilities while reducing cost and complexity.

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