Attitude Dynamics and Control of a 3U CubeSat with Electrolysis Propulsion

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Electrolysis propulsion systems at the CubeSat scale combine the best elements of liquid chemical and electric propulsion systems to create a system that is well-suited for this scale, providing over 800 m/s ΔV while conforming to standard CubeSat specifications. The operations concept for the proposed spacecraft architecture is based on passive spin stabilization, and several factors that determine the required spin rate are investigated. Actuation strategies for achieving this spin through the use of magnetic torquers are discussed. Issues involving the dynamics of a spinning 3U mission, such as gravity gradient torques, shifts in the center of mass due to propellant consumption, and precession of the spacecraft are analyzed. Synergistic design of the propulsion subsystem and the attitude-control concept leads to an efficient, low-risk, high-performance architecture.

Nomenclature

ΔV = change in orbital velocity
Bo = Bond number
Δρ = density difference
a = acceleration
σ = surface tension
L = characteristic length
ω = spacecraft angular velocity
r = position of spacecraft center of mass relative to Earth’s center
τGG = gravity gradient torque
μ = Earth’s gravitational parameter
I = spacecraft inertia dyadic
τM = torque produced by magnetorquers
M = magnetic moment
B = magnetic field
θ = precession angle
ν = true anomaly

I. Introduction

ELECTROLYSIS propulsion presents a novel option for propulsion at the CubeSat scale. This types of propulsion system combines the best characteristics of chemical and electrical propulsion technologies to create an architecture that, at the CubeSat scale, results in reliable, safe operation and high ΔV.¹ The 3U CubeSat proposed in previous papers as a demonstration mission for the technology raises a number of flight-dynamics issues, which require further analysis if such a technique is to be implemented.² This paper addresses the key issues of attitude dynamics and control in an effort to establish a baseline operations concept for a successful CubeSat mission.

Electrolysis propulsion systems have been proposed and studied for large spacecraft in the past,³ ⁴ ⁵ but no flight tests or prototypes are known to have flown. These earlier systems feature a complex architecture involving several gaseous storage tanks, which causes the system mass to be prohibitive.⁶ The architecture proposed here takes advantage of the small size and limited mass of the 3U CubeSat bus to motivate the design of a propulsion system

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that, while similar in the fundamental operation to large electrolysis systems, is simpler and much better suited to the CubeSat scale. The system works by converting solar energy into chemical energy, which can be used to propel the spacecraft. The electricity generated by the solar cells is used to electrolyze liquid water into hydrogen and oxygen gas. Effectively, the propellant itself serves as a battery, storing the electrical power with far greater mass efficiency and none of the electrical losses associated with batteries. The mass savings due to eliminating batteries for the propulsion system and replacing that mass with propellant is a striking benefit of this architecture.

When the gas pressure is sufficiently high and a burn is desired, a solenoid valve opens to allow gas to flow into a combustion chamber. The flight software then commands a spark that ignites the gas mixture. The combustion products then expand through a small nozzle located near the spin axis, generating approximately 5N of thrust and approximate impulse of 0.6 Ns per pulse.

The spacecraft electrolyzes water only while it is in sunlight and then combusts the gases infrequently—for example, once per orbit in the case of orbit-raising— with each burst lasting less than a second. This approach helps ensure that enough gas has accumulated for a successful firing but also limits demands on the design of the electrical and power subsystem. The satellite’s orbit is also raised efficiently, since thruster impulses occur only at perigee in such a case. The process repeats, each time combusting about 1 g of water.

This paper describes a candidate design, in which the electrolyzers to be used are composed of platinum mesh anode and cathode, separated by a thin layer of a proton exchange membrane. The electrodes are within the liquid water tank, which contains distilled water. The proton-exchange membrane allows for protons, but not electrons, to pass between the two electrodes. When powered, oxygen and hydrogen gas forms on the electrodes. This gas is kept as a mixture, not separated into hydrogen and oxygen tanks. Each of the electrolyzers operates at the same voltage. Several of the electrolyzers are to be used in parallel so that as much power as possible goes toward electrolysis of the water. The number of electrolyzers that are operational at any point determines the power devoted to electrolysis.

The spacecraft spins in order to separate the water from the oxygen and hydrogen mixture in the fuel tank, ensuring that the valve leading to the combustion chamber always ingests gas, not water. This spin axis is aligned, as close as possible, with the thrust axis, as shown in Figure 2. The spin is established after spacecraft separation from the P-POD deployer by magnetic torquers embedded in the solar panels. The spin rate must be high enough to both effectively separate the water from the electrolyzed gases and provide gyroscopic stability in the presence of disturbance torques. In this paper, precession caused by the gravity-gradient torque and by misalignments between the thrust axis and the spin axis is analyzed.

The proposed sequence of events for this CubeSat begins in Geostationary Transfer Orbit (GTO). Perigee burns raise apogee quickly, making the resulting orbit changes much more dramatic than a sequence starting in Low Earth Orbit (LEO). These burns produce more readily measured altitude changes, which makes for a more effective technology demonstration. The spacecraft is estimated to have enough propulsive capability to reach Earth escape from GTO, enabling a lunar flyby or interplanetary missions, among other applications. However, the initial GTO launch introduces some complications. One is that the initial spin-up suffers from the fact that magnetic actuators are less effective when the spacecraft is far from Earth. Section IV of this paper explores this aspect of the control architecture. It discusses the use of magnetic torquers as applied in two phases of the mission: the stabilization maneuvers starting from an unknown spin state after separation, and the establishment of the desired spin state of the spacecraft.

Figure 1. Cutaway View of 3U Electrolysis Propulsion CubeSat. The satellite’s major components are highlighted. Body coordinates are shown in green.
II. Satellite Dynamics and Attitude Requirements

A. Post-separation stabilization

The nature of the P-POD deployer prevents the spin state of the satellite at separation from being specified. It most likely will be a slow-speed spin with some nutation. In any case, it will not be close to the desired final configuration of high-speed, major-axis spin. The candidate spacecraft design includes magnetic torquers in the solar panels that are used to spin up the spacecraft about the thrust axis. Simple on-per-rev pulses from the magnetic torquers result in some undesirable nutation. The solution, nutation damping, consists of reducing the transverse rate—that is, the component of the angular-velocity vector transverse to the desired spin axis. While it would be possible to use the magnetic torquers in a more subtle way to damp nutation as well as alter spin speed—i.e. three-axis rate control—this added subtlety turns out to be unnecessary. The spacecraft is designed such that the major axis of inertia is parallel to the thrust axis, making the desired spin a stable state in the presence of energy dissipation. Since a large fraction of the spacecraft’s initial mass (25%) is the water in the fuel tank, nutation is expected to damp passively, and quickly. The spacecraft will naturally tend towards a flat spin about the major axis of inertia, keeping the thrust axis aligned with an inertial direction. In fact, either a positive or a negative spin about this axis is acceptable, which makes for a robust spin-stabilization design.

Through the arrangement of internal components, the spacecraft’s principal moments of inertia establish the thrust axis as a decidedly major axis, at least 1.2 times greater than the other two principal axes. While this arrangement ensures that the spin state at separation would naturally evolve towards the desired thrust-axis spin, active control of the spin axis is necessary in the initial stages of the mission so that the desired inertial angular-momentum vector can be achieved. Most propulsive burns would occur at perigee in order to raise the spacecraft’s orbit as much as possible. In order for the burns to be most effective, the spacecraft’s angular-momentum vector, and therefore its thrust vector in equilibrium spin, must be aligned with the orbit velocity at perigee. The combination of magnetic torquers and passive damping achieve this alignment to the extent that the principal axis aligns with the thrust axis.

B. Thrust axis spin

A constant spin about the thrust axis separates the liquid water from the electrolyzed gases. This spin field obviates the need for the complex multi-tank storage systems proposed for large scale electrolysis propulsion systems and allows the CubeSat system to function with just one tank. This spin field is set up after the spacecraft’s attitude and spin rates have been established in the initial magnetorquer maneuver. Before any maneuvers can be performed, the spacecraft must have reached a steady spin, in order to avoid the ingestion of liquid water in the valves. Between pulses, the spacecraft would have an entire orbital period (10.6 hours for the initial GTO) to dissipate any nutational disturbances caused by the thrust pulse. During this time, the magnetic torquers can also be used to precess the momentum vector to align the thrust axis for subsequent burns, if necessary.

The choice of spin rate is driven primarily by the Bond number ($Bo$) of the fluid. This nondimensional parameter measures the relative importance of surface tension effects and inertial effects:

\[
Bo = \frac{\Delta \rho \ a \ L^2}{\sigma}
\]  

where $\Delta \rho$ is the difference in density between the water and the gas mixture, $a$ is the magnitude of the acceleration due to the spacecraft spin, $L$ is a characteristic length scale, and $\sigma$ is the surface tension of the liquid. In the case of the 3U spacecraft described, the acceleration is approximately

\[
a = \omega^2 r
\]

where $\omega$ is the spacecraft’s spin rate, and $r$ is the distance of the fluid’s free surface from the center of mass of the spacecraft. In order for the inertial effects to dominate over the surface tension effects, and therefore for the gas
bubbles to aggregate towards the inboard side of the tank, $Bo$ should exceed unity. However, in order for inertial effects to be decidedly dominant, the goal is for $Bo$ to be an order of magnitude larger, around $10^{7,8}$. The minimum spacecraft spin rate is therefore

$$\omega = \left( \frac{Bo \sigma}{\Delta \rho \tau L^2} \right)^{\frac{1}{2}} \quad (3)$$

The distance from the center of mass of the spacecraft to the fluid free surface is between 0.05 m and 0.15 m, depending on the fill fraction, and the characteristic length is taken to be 0.1 m, the width of the tank. The density difference, $\Delta \rho$, between water and a stoichiometric mixture of hydrogen and oxygen gas can be approximated by the density of water (998.6 kg/m$^3$), since even at the 10 bar pressure inside the tank, the density of the gas mixture is about two orders of magnitude lower than the density of water. The surface tension of water, $\sigma$, is 0.0728 N/m. The resulting spin rate necessary to attain a minimum Bond number of 10 would then be 1.21 rad/s, or about 11.5 RPM.

The thrust-axis spin also has the important advantage of providing passive robustness to external torques and misalignments of both the jet of water vapor and mechanical features. For typical payload configurations, the center of mass shifts by an estimated 0.032 m during the spacecraft’s lifetime. This shift takes place because the propellant tank is located entirely on one side of the satellite, and as the propellant is expended, the satellite’s mass distribution changes. The engine is aligned so that the thrust vector passes through the midpoint of the CM’s path. This tactic minimizes the overturning torque throughout the mission, also reducing the need for reorienting the momentum vector to align with the velocity vector. Judicious timing of engine firings or simply a fast-enough rotation of the satellite can mitigate both the nutation and unwanted precession of the momentum vector.

III. Analysis of Center of Mass Shift Throughout the Mission

As the semimajor axis of the orbit increases due to the thrust maneuvers, propellant is expended. This effect influences the rotational dynamics of the spacecraft and it can have a small impact on the direction of the net $\Delta V$.

A. Calculation of CM shift

The initial center of mass of the spacecraft can be estimated by making several simplifying assumptions. The spacecraft’s structural mass is estimated to be 0.5 kg, and is taken to be uniformly distributed amongst the three 1U segments, not contributing to a shift in CM in either direction. The propellant tank contains approximately 1 kg of water, distributed over the +X 1U segment. The electronics and payload package on the -X 1U segment is taken to have a mass of 1 kg. The remaining mass is that of the propulsion system, which is placed in the central 1U segment and can be shifted slightly so that the thrust axis is within a few centimeters of the geometric centroid of the CubeSat. Based on this simple model, and assuming the origin is at the geometric center of the satellite, the center of mass of the satellite is at $x = 0.0143$ m at the beginning of the mission, and migrates to $x = -0.02$ m when the tank is empty. The total shift in the center of mass is therefore 0.0343 m. The ideal placement in the $x$ direction for the thruster is then 0.29 cm from the geometric center of the satellite, in the direction away from the propellant tank. The moment arm for thruster-induced torque on the spacecraft is greatest at the start and at the end of the mission, and is at most 1.71 cm.

B. Effect of CM Shift on Spin Axis Precession

The torque imparted on the spacecraft due to a thrust-axis misaligned with respect to the spin axis causes the spacecraft’s angular momentum vector to precess. The spin axis is at a maximum of 1.71 cm from the thrust axis, and as explained above, is closer than that for most of the mission. The thrust is approximately 5 N, and each burst lasts approximately 0.5 s. The torque imparted on the spacecraft when the moment arm is largest has a magnitude of 0.855 Nm, and is in the $y$-direction in the body-fixed coordinates. For the worst case of an instantaneous thrust impulse, the change in angular momentum is

$$\Delta h = \tau \Delta t \quad (4)$$

where $\tau$ is the torque on the spacecraft and $\Delta t$ is the time during which the pulse is applied. In this case, the total change in angular momentum is 0.0428 Nms. The angular momentum of the spacecraft due to the spin calculated in Section II above is

$$h = I \cdot \omega \quad (5)$$
where $I$ is the spacecraft’s inertia dyadic. For a uniformly-distributed, 4kg, 3U CubeSat, the principal moments of inertia are as indicated in Table 1. For the propulsive 3U CubeSat, the principal moment of inertia aligned with the thrust axis, $I_{zz}$, is approximately 1.2 times larger than $I_{yy}$, in order for the spin about the thrust axis to be stable. For this analysis, however, the uniformly-distributed CubeSat model is used. $\omega$ is in the z-direction in body coordinates. We will consider the case of no products of inertia; i.e., the inertia matrix is diagonal in body coordinates. In that simple case, $h$ is also in the z-direction and has a magnitude of 0.0403 Nms. Since the torque and the momentum vectors are orthogonal, the change in angular momentum tilts the momentum vector by an angle

$$\theta = \tan^{-1}\left(\frac{\Delta h}{|h|}\right)$$  \hspace{1cm} (6)

For the rotation speed calculated in Section II, the precession angle is 46.7 degrees, an unacceptably large precession angle. In order to reduce this angle to a more manageable one, the rotation speed must be higher than the minimum required for an adequate $Bo$. Rearranging Eqs. 4-6 yields the rotation speed in the z-direction as a function of the desired maximum precession angle:

$$\omega_z = \frac{\tau \Delta t}{I_{zz} \tan(\theta)}$$  \hspace{1cm} (7)

For example, a rotation speed of 4.8 rad/s would produce a precession angle of 15 degrees. For an angle of 10 degrees, the rotation speed would have to be 7.4 rad/s. However, for higher spin speeds, the approximation that the thrust pulse is instantaneous becomes less accurate. If the torque is applied for a significant fraction of the spin period, the torque direction constantly changes, further reducing the precession angle. If the spacecraft were to complete an entire rotation in the time it takes to pulse the thruster, then the effects of that torque on the spacecraft’s spin axis would cancel. Therefore, the assumption of impulsive torque represents a bounding case. The actual duration of the impulses varies between 0.25 s and 0.5 s, which would represent at least 30% of an orbit period.

### C. Effect of Products of Inertia on Spacecraft Nutation

Besides the effects of the center of mass motion, an uneven mass distribution causes the inertia matrix to contain cross terms, or products of inertia, which introduces wobble into the spacecraft’s motion (a constant tilt, as seen in the body axes). When products of inertia are present in the inertia tensor, the spacecraft’s angular momentum vector is not aligned with the desired spin axis in equilibrium, and therefore the thrust pulses can cause the spacecraft to further precess if the wobble complements some other, existing angular bias.

A model of the CubeSat with all components needed for operation has been created in SolidWorks. Analysis of the products of inertia estimated using the model provides approximate values of the deviation between the spacecraft’s principal axes and its body axes. The $\hat{z}$ axis as shown in Figure 1 deviates by approximately 9 degrees from the principal axis. The deviation in the body axes are shown in Table 2. This discrepancy can be mitigated in several ways. The analysis of the angles between the two axes can be used to shift the placement of internal components such as avionics boards, antennas or batteries. However, it is likely that the mass properties of the spacecraft’s components are not accurate enough for this approach to be completely effective. Small ballast masses can also be placed to help balance the CubeSat. There is enough expected margin in the mass budget to allow for a moderate amount of ballast. Another approach would be to align the thruster’s axis of symmetry as closely as possible with the estimated principal axis. A combination of these methods could be used in the final design and integration steps of the CubeSat, minimizing the impact of products of inertia in the body-frame inertia tensor. However, the high-speed spin tends to cancel out these imbalances, so that a long-enough pulse may be nearly centered on the spin axis, with only a cosine loss in impulse due to wobble.

### IV. Magnetic Actuation

Spinning the 3U spacecraft is a crucial part of the proposed technology-demonstration mission and must be accomplished using the magnetic torquers embedded in the CubeSat’s solar panels. Magnetic actuation is used by many CubeSats in LEO. However, the use of magnetic torquers in GTO presents a few complications. Earth’s
magnetic field drops as $1/r^3$, where $r$ is the distance from Earth’s center. A GTO orbit would take the spacecraft from LEO altitude, approximately 7,000 km from Earth’s center, to GEO altitude, 42,160 km away. The magnetic field is 218 times weaker at GEO. The applicable torque, which is proportional to the magnitude of the magnetic field and the magnitude of the magnetorquer’s magnetic moment, would therefore decrease just as much. Therefore, in the initial GTO orbit, the magnetorquers will only be able to effectively torque the spacecraft during a fraction of the orbit near perigee. Since the velocity of the spacecraft is higher when near perigee, the time spent in the portion of the orbit where actuation is possible is small. The torque applied by a magnetorquer is

$$
\tau_M = M \times B \tag{8}
$$

where $M$ is the magnetorquer’s magnetic moment and $B$ is the Earth’s magnetic field. The magnitude of the torque can be estimated by using a dipole model of the Earth’s magnetic field and the approximate values of magnetic moment of the magnetorquer coils. At perigee, the maximum magnitude of available torque is $|\tau_M| = |M| \cdot B_0 \left( \frac{R_E}{r} \right)^3$, where $B_0$ is the magnitude of Earth’s magnetic field at the magnetic equator. For a typical magnetorquer embedded in the solar panels of the CubeSat, magnetic moments are approximately 0.13 $Am^2$ for a 3U panel and 0.038 $Am^2$ for a 1U panel. This combination yields a maximum torque of $4.04 \times 10^{-6} \text{ N}$. At a spin speed of 7 rad/s, it would take an actuator applying this maximum torque 16.7 minutes, a small fraction of a GTO orbit, to reorient the spacecraft 1 degree. In order to command a control torque on the spacecraft, the magnetorquer is given a current command. Eq. 8 can be used to solve for the magnetic moment by taking the cross product of the magnetic field $B$ with Eq. 8 and then expanding the right hand side using the triple product identity.

$$
B \times \tau_M = B \times (M \times B) = B^2 M - (B \cdot M)B \tag{9}
$$

Because components of the magnetic moment in the $\hat{B}$ direction do not contribute to torque, $B \cdot M = 0$, and Eq. 9 reduces to

$$
B \times \tau_M = B^2 M \tag{10}
$$

The magnetic moment required to produce a given control torque is therefore

$$
M = \frac{B \times \tau_M}{B^2} \tag{11}
$$

This magnetic moment is produced by applying a voltage on the looped traces of known area embedded in the solar panels, which causes a current that generates the magnetic moment. The voltage required can be found from the pseudoinverse of a matrix representing the area of each loop and its normal vector.

### A. Numerical simulation of magnetorquer actuation

A simulation of the attitude dynamics of the satellite has been implemented in Simulink. The simulation includes closed-loop control of the satellite based on magnetorquers embedded in the solar panels, as described in Eqs. 8-11. The closed-loop control is implemented as a PD controller. The magnetic moments used in the simulation are those produced by the 1U and 3U solar panels available from GomSpace. The simulation starts at perigee and propagates the orbit and attitude of the satellite subject to magnetic control torques. The magnetic field model used is the 1995 International Geomagnetic Reference Field (IGRF). The satellite’s orbit is a geostationary transfer

![Figure 3. Simulation of satellite de-spin maneuver using magnetorquers.](image-url)
orbit with perigee at an altitude of 700 km.

The simulation was repeated 100 times in a Monte Carlo analysis, each time starting from a random spin state meant to simulate the uncertainty in the spin rate after separation from the P-POD. In all cases, the spin rate in each axis had a magnitude of at most 0.5 rad/s, several times larger than the likely separation spin rate of 7 deg/s (0.122 rad/s) \(^\text{12}\). The controller then attempted to reduce the spin rate to zero in as little time as possible, taking into account the hardware limitations in the form of actuator saturation. Results from these simulations are shown in Figure 3. The first plot shows the component of the angular-velocity vector about the axis of maximum inertia for all 100 runs of the simulation. Because of the liquid damping, which is implemented in the simulation as a Kane damper \(^\text{13}\), the satellite stabilizes into a major axis spin within the first orbit. While this is approximately the behavior expected, it should be noted that the simulation is not meant to model the liquid motion exactly; such a model would require extensive test data to validate the complex liquid behavior. \(^\text{8}\) Instead, simple damping is meant to capture the main effect the liquid will have on the spin state of the spacecraft. The nutation-damping time constant for this simple model can be matched to test data to represent a particular flight configuration.

The second plot shows the strength of the magnetic field at the satellite’s position throughout the simulation. Because the actuators can apply only limited current, the strength of the magnetic field limits the maximum torque that can be applied. The local magnetic field is much stronger when the spacecraft is close to perigee and drops off approximately as \(r^{-3}\). Therefore, the majority of the change in spin rate occurs when the spacecraft is near perigee. This fact can be used to implement a more sophisticated control algorithm that only actuates when the spacecraft is within some angular distance of perigee, if necessary.

The final spin rates represented in principal axes for the 100 runs of the simulation are shown in Figure 4. The controller is able to keep the final rotation rate to within \(5 \times 10^{-4}\) rad/s about all three axes.

**Figure 4.** Final angular velocity of the satellite about each principal axis.

**B. Post-separation stabilization**

The de-spin maneuver simulated above represents the initial attitude maneuver that needs to be performed to eliminate the angular velocity imparted during separation from the P-POD. The exact number of orbits needed to complete this maneuver depends on the spin state of the spacecraft when it separates. The maximum expected value of spin rate after separation is 0.122 rad/s, as used in the simulation above. However, the spin rate of a previous CubeSat mission with unexpectedly large spin rate at separation was about 5.6 rad/s. \(^\text{14}\) Reducing this rotation rate to zero could require up to 60 orbits. However, because the CubeSat contains liquid water in the propellant tank, liquid damping tends to bring the spacecraft into a stable spin about its axis of greatest inertia. If the spacecraft happens to be deployed with a similarly large rotation rate, it will transition into a major axis spin, which in some cases can allow the spacecraft to transition directly into the thrust axis spin mode by either increasing or decreasing its spin rate to 7.4 rad/s. A decision to follow that branch in the concept of operations would be made in realtime.

**C. Actuation for thrust axis spin**

The maneuver to spin up to the final desired rotation speed begins once the spacecraft’s post-deployment orientation and angular velocity have been established. The spin-up maneuver can be done over several orbits and
ends with the spacecraft spinning at the desired rate of 7.4 rad/s and aligned such that the spin axis is parallel to the spacecraft’s orbital velocity vector at perigee.

Throughout this setup phase, the CubeSat is not electrolyzing water and can therefore use the power normally associated with electrolysis to actuate the magnetorquers. Operating all six of the magnetorquers at once would require at most 1W, power which is readily available when not electrolyzing. The amount of time and energy necessary to achieve the desired spin state using only the magnetorquers embedded in the solar panels can be estimated from the simulations presented above. Starting from rest, the spacecraft would require about 90 orbits to reach the final spin rate of 7.4 rad/s. This scenario represents a worst case, and if the mass budget and volume allow, an external magnetorquer rod can be included in the CubeSat. Commercially available magnetorquer rods have masses of 30 to 50 g and magnetic moments of approximately 0.2 Am², which would almost double the torque available about one of the axes. This option would roughly halve the time the CubeSat spends in its initial spin-up phase and would be appropriate for missions that require a faster initialization.

V. Analysis of Gravity Gradient Torques on Rotating Satellite

Environmental disturbance torques that act on the 3U CubeSat can cause the spin axis to precess, leading to an undesired thrust-vector direction. A high enough spin rate can stiffen the spacecraft’s response to the point where it mitigates any disturbance torque. If the disturbance torque cannot be rejected in this way, it must be compensated by the magnetorquers in order to keep the spin axis properly aligned. Assuming that the geometric center of the spacecraft is coincident with the center of mass, the torque due to gravity gradient effects is approximately

$$\tau_{GG} = \frac{3\mu}{r^3} [\hat{r} \times (I \cdot \hat{r})]$$

where $\tau_{GG}$ is the gravity gradient torque, $I$ is the inertia dyadic for spacecraft, $\mu$ is the Earth’s gravitational parameter and $\hat{r}$ is the spacecraft’s position from the Earth’s center. The worst-case torque occurs when the spacecraft is closest to the Earth, at perigee. As the spacecraft orbits, its long axis (x-axis in body coordinates) is not aligned with the radial direction in the Earth’s frame. The direction of the resulting torque, however, changes throughout the orbit, as is evident in Figure 5. The torque through an entire circular orbit would average to zero. However, for a GTO orbit, the orbit-averaged torque would not necessarily average to zero. The magnitude of the torque depends on both the distance from the center of Earth and the orientation of the CubeSat. Since in this case the CubeSat’s x-axis must be aligned with the radial direction at perigee (in order for the thrust pulses to be in the velocity direction) the torque does average out to zero over one orbit. It is, nevertheless, important to quantify the precession caused by the periodic torque. This can be done by applying Eq. 12 over one quarter of the orbit. The largest torque will occur in the interval between $\nu = 0$ and $\nu = \frac{\pi}{2}$ (as well as in the interval between $\nu = \frac{3\pi}{2}$ and $\nu = 2\pi$) since it is in those intervals when the spacecraft is closest to Earth. The largest magnitude of torque experienced in this interval is $3.45 \times 10^{-8}$ Nm. The time spent in the interval is 1675 s when the spacecraft is in its initial GTO orbit. A simple worst-case analysis would then mean that the total change in angular momentum would be given by Eq. 4. In this case, the total change in angular momentum would be $5.78 \times 10^{-5}$ Nms. The angular momentum of the spacecraft due to the spin is calculated in the section above according to Eq. 5. The gravity gradient torque in this case would be orthogonal to the z-direction, which

![Figure 5. Gravity Gradient Torque for One Orbit. The direction and magnitude of the gravity-gradient torque change throughout the initial orbit. Torque is in the direction normal to the page, Angular momentum vectors shown in red.](image-url)
causes the angular momentum vector to precess. The degree to which the angular momentum vector tilts is given by Eq. 6, which for the slower spin rate calculated in section II would be 0.0014 rad, a very small deviation. A numerical analysis taking into account both the spacecraft’s varying velocity and the varying torque in the portion of the orbit between \( \nu = 0 \) and \( \nu = \frac{\pi}{2} \) yields even less precession angle, 0.0007 rad.

VI. Conclusion

This demonstration mission is designed to test an electrolysis propulsion system. The spacecraft benefits from an initial GTO orbit, requiring less than 800 m/s for Earth escape. This system provides that much impulse and takes advantage of time-honored principles of spinning dynamics to simplify the design of the propulsion system. Specifically, in order to separate the gases from the liquid, the satellite rotates about its thrust axis. Without this spin, the satellite would be able to fire its propulsion system only with more complex propellant management. The spin also provides gyroscopic stiffness and robustness to misalignments and torques. The minimum spin rate needed to sufficiently separate the liquid propellant from electrolyzed gases was calculated based on the fluid’s Bond number and revised based on an analysis of the precession caused by thrust pulses in the presence of a displaced center of mass. This spin speed is readily achieved.

Magnetic torquers embedded in the solar panels, coupled with passive fluid-damping effects, regulate the spacecraft’s attitude and properly orient the spacecraft after separation from the launch vehicle. Once the spacecraft is in a stable spin about the major axis of inertia, these torquers are used to properly align the spacecraft in its orbit and to bring the spacecraft up to its final spin rate. A numerical simulation of post-separation spin maneuvers is presented. The effect of gravity gradient torques on the spinning spacecraft is analyzed, and it is shown that the precession caused by these torques is not significant. A similar approach can be taken in order to analyze other disturbance torques that might be present.

This study provides a basis of an operations concept and an attitude control system for the proposed 3U electrolysis propulsion system demonstration mission. Future work towards this goal includes the orbital trajectory design for a reference mission utilizing pulsed propulsion, determination of the sensing capabilities of magnetometers available at this scale, and analysis of the thermal environment within the satellite due to solar heating. A key issue to be addressed is the requirement to maintain the water in liquid form. Spinning is expected to help here as well, providing a rotisserie effect for somewhat uniform heating.

Electrolysis propulsion systems are designed for the CubeSat scale and with the goal of making orbits beyond LEO more accessible to scientific investigation via CubeSats. A successful mission demonstrating the high-\( \Delta V \) capabilities of this propulsion system would greatly increase the viability of future CubeSat missions beyond low earth orbit.

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