Quantum Cryptography
Between Science and Business

Wilke Castelijns
Maria van Rooijen
David Venhoek
Patrick Uiterwijk
Preface

In this report we discuss the results of our project for the Radboud Honours Academy, concerning Quantum Key Distribution (QKD). We originally started this project with a much larger research subject in mind, namely quantum cryptography and quantum computers. At the beginning of this year, none of us had even heard of quantum cryptography. It was professor Klaas Landsman who introduced this as a research subject for the first year of the Honours programme. But after a short time, we felt that narrowing this subject was necessary: we decided to forget about quantum computers and focus on currently commercially available systems implementing QKD protocols. In particular, our goal was to investigate the business versus the hacking culture. By exploring both the suppliers of quantum cryptographic devices and the people who want to hack these systems, we wanted to get a complete and realistic a picture of the physical implementations of quantum cryptography that are nowadays possible.

The first part of our work consisted of theoretical research. We had to lay a mathematical foundation for our project and get a proper understanding of quantum mechanics and QKD protocols. We visited Serge Fehr at the CWI\(^1\) and Barbara Terhal at RWTH (Aachen University). They answered our theoretical questions and helped us finding the right people for the remaining and most important part of our project, namely the physical implementations of QKD and their weaknesses. During our trip to Züirich, we visited Normand Beaudry at the Eidgenössische Technische Hochschule (ETH) Züirich, who already told us a lot about QKD protocols used by the company ID Quantique and their physical implementations. The day after, we went to Geneva, where we visited ID Quantique and GAP-Optique. At ID Quantique, one of the few companies in the world that produce quantum cryptographic devices, Damien Stucki showed us their quantum key distribution products and explained how they work. At GAP-Optique, the Group of Applied Physics at the University of Geneva, Nino Walenta showed us their lab and explained more about QKD protocols. To discover the weaknesses of the ID Quantique devices and learn more about possible attacks, we made a trip to Germany. In München we visited the Ludwig-Maximilians-Universität, where Sebastian Nauerth and his colleagues told us about free-space QKD and showed us many quantum cryptography related experiments. In Erlangen we visited Nitin Jain at the Max Planck Institute, who told us a lot about attacks on commercially available QKD systems. Finally, we contacted the famous ‘quantum hacker’ Vadim Makarov, who was very helpful in answering questions concerning the future of QKD. Unfortunately, we were not able to visit him (as we originally intended), since he recently moved from Trondheim to Waterloo in Canada.

\(^1\)Centrum voor Wiskunde en Informatica, www.cwi.nl
After all these visits and extensive literature study, we think we are able to give a realistic overview of commercially available QKD systems and possible attacks exploiting their practical weaknesses, which we tried to do in this report.

In what follows we will first give a brief description of cryptography and quantum-cryptography. Also, we will give a short overview of the quantum-mechanic theory required for quantum cryptography. Subsequently we focus on commercially available product that implement quantum cryptography. In particular, we explore the gap between their theoretical security and their practical vulnerability.

We would like to thank all the people we visited for giving us so much of their time and being almost as enthusiastic about our project as we were ourselves. We also want to thank the Radboud Honours Academy for giving us the opportunity to make our trips to Switzerland and Germany. Finally, we want to express our thanks to professor Klaas Landsman, who has been a great and involved supervisor throughout this year.

Figure 1: München, April 25, 2012. From left to right: Wilke, Patrick, Maria, David
## Contents

1 Introduction to cryptography ................................. 5  
   1.1 What is cryptography? .................................. 5  
   1.2 Why cryptography? ..................................... 5  
   1.3 Types of cryptography ................................... 5  
   1.4 Methods of cryptography ................................ 6  
   1.4.1 Example: RSA ....................................... 7  
   1.5 The problem with classical cryptography .............. 7  

2 A very brief introduction to quantum mechanics .............. 9  
   2.1 States .................................................. 9  
   2.2 Superposition .......................................... 10  
   2.3 Measurements .......................................... 10  
   2.4 Combinations of systems ................................ 11  
   2.5 Transformations ........................................ 12  

3 Quantum key distribution protocols .......................... 13  
   3.1 Qubit based ............................................ 13  
      3.1.1 BB84 ............................................. 14  
      3.1.2 Ekert 91 .......................................... 14  
      3.1.3 SARG04 ............................................ 15  
   3.2 Continuum based ........................................ 15  
      3.2.1 Coherent one way .................................. 15  

4 Post-processing ............................................... 17  
   4.1 Error correction ........................................ 17  
   4.2 Privacy amplification ................................... 17  

5 Proving security .............................................. 19  
   5.1 Model .................................................... 19  
   5.2 Security of QKD in the model ............................ 19  

6 Commercially available products .............................. 21  
   6.1 ID Quantique SA ........................................ 21  
   6.2 Specifications of Clavis²  ................................ 22  
      6.2.1 Software ........................................... 23
6.2.2 Physical implementation

7 Weaknesses of QKD devices
7.1 Preparation
7.2 Channels
7.3 Detection
    7.3.1 Avalanche photo-diodes
    7.3.2 Superconducting nanowire single photon detectors
    7.3.3 Faked states

8 Possible attacks
8.1 Intercept-resend attacks
    8.1.1 Faked states attack
    8.1.2 Phase-remapping attack
8.2 Photon number splitting attack
8.3 Time-shift attack
8.4 Quantum blinding
    8.4.1 Continuous-wave blinding
    8.4.2 Thermal blinding

9 Summary and conclusion

10 Looking forward

Appendix A. Internal structure of Clavis²

Appendix B. Components of the physical implementation

Bibliography
Chapter 1

Introduction to cryptography

1.1 What is cryptography?

“Cryptography” is a word that comes from the contraction of the Greek words κρυπτός (kryptós) for “hidden”, and γράφω (gráfo) for “writing”. Literally translated, it says “hidden writing”, which describes the purpose of cryptography quite well: “writing”, or encoding, a piece of data or a text such that only the intended recipient can read the “clear text”, and anyone else gaining a copy of the “cipher text” has no clue of what it says, unless they have the required key to decrypt it.

1.2 Why cryptography?

There are many reasons why one would use cryptography. For example, in the case of submarines: communication between the submarine and the coast would have to be secured from the ears of the enemy, because otherwise the enemy would know exactly when and where a submarine would strike, and so they could intercept them.

Another, current, example is banking on the internet. If anyone would be able to eavesdrop on all data sent between you and your bank, they could empty your bank account.

1.3 Types of cryptography

Currently there are two main types of cryptography.

The first type of cryptography is symmetrical, or shared-key cryptography, where both parties have the same key that is used to both encrypt and decrypt the data. An example of this is the substitution cipher where you just rotate all letters by 13 positions in the alphabet. This means that if you use the same procedure twice on a text, you recover the original text.

The other, much more recent, type is asymmetrical-, or private-key cryptography, where each participant has a personal key pair consisting of a public and private key. The idea is that one can publish one’s public key, which everyone can subsequently use
to encrypt a message sent to him. But one would need the corresponding private key to decrypt the message, which only the key holder should possess.

The most popular type is symmetrical cryptography, since this requires only one key for both parties, instead of two public and two private keys. However, with symmetrical cryptography, one has the problem of key exchange. The problem is that one will need to share a big key with the other participant with whom one would like to communicate, but will need to agree on the used key securely, because if an eavesdropper learns the key, the participants still lose all secrecy of their messages. Currently, the standard solution for this is that one party generates the key, encrypts it asymmetrically with the public key of the other participant, and finally sends it to him. However, this also means that when the asymmetrical algorithm that has been used for the key exchange is broken, the shared key again becomes “public”.

1.4 Methods of cryptography

There are various ways to encrypt data, that each have their own advantages and disadvantages. A scheme to encrypt data is called a “cipher”.

One of the first known and used ciphers is the “substitution cipher”, where one replaces each letter of the alphabet in the plaintext by a specific other letter to produce the ciphertext. Stipulating which letter is replaced by which provides the key for this cipher. An example could be to replace each letter by the next letter in the alphabet, so one would replace $a$ by $b$, $b$ by $c$, and so on. To decrypt, the receiver would have to replace each letter in the received text by the letter preceding it in the alphabet. This is a very simple cipher, and it can easily be broken if one just tries out all the possibilities (brute-forcing) of the substitution, or if one performs frequency analysis of letters in the ciphertext.

Since the previous procedure is a very easy method of encrypting data, it is also easy to break, and hence some adaptations have been invented that were a bit more secure. One such example is the Enigma cipher used by the Nazi’s, where each letter is still replaced by another letter in the alphabet, but the letter by which it gets replaced is not constant. This could be done because the Enigma used three or four rings which guide the electric signal produced by a press on a key on its keyboard to a specific letter in the end result, after which the rings turn a bit like a clock. This would be very hard to crack without having such a device, because one would need to know the configuration of the rings, as well as the initial position when the encryption started: frequency analysis would be impossible, because the letter that replaces another letter is not fixed. But brute-forcing combined with educated guesses is still an option, if one has enough ciphertext-examples, and this is indeed the way the Enigma cipher was broken during WWII.

A third way of “encrypting” data is called “steganography”, where one hides the data one wants to send inside other data, for example a movie or a picture. One way to do this is by just replacing the last bit of each byte in a picture by the next bit in the source document. If one would just look at a picture where steganography is used to hide a document, one would not notice too much difference, because those low bytes are only used for detailed color data. However, if one knows that the picture contains hidden data, one could retrieve it by extracting the last bits of each byte in the received picture.

One of the most basic digital ciphers is the one-time pad cipher, which works by executing the bit-wise XOR between the message you are trying to send and the key
stream. If you do not repeat the key-stream, this cipher is unbreakable for anyone not having the key.

But currently, we use ciphering schemes that are more complex, based on a mathematical function which takes the plaintext as argument, and gives the ciphertext as its result but which is practically impossible to invert. Some of the most often used such schemes are RSA, Rijndael (AES) and Diffie-Hellmann.

1.4.1 Example: RSA

We will give an example of a contemporary asymmetrical encryption cipher, in the form of the RSA\textsuperscript{1} cipher, and explain what part of it is the most crucial.

In this example, Alice will create a key pair \((K_{\text{public}}, K_{\text{private}})\).

First, Alice randomly chooses two big prime numbers \(p\) and \(q\). Then she needs to compute \(n = p \times q\), which is one part of the public and private keys. Then she computes \(\phi(n) = (p - 1)(q - 1)\). Next, she needs a number \(e\) that has the following properties: \(1 < e < \phi(n)\) and \(\gcd(e, \phi(n)) = 1\). This \(e\) is very often a fixed number, for example, most software implementations just use \(e = 65537\), because it is harmless if a lot of people use the same value. Next, she calculates \(d = e^{-1} \mod \phi(n)\).

Now the public key of Alice is \((n, e)\). This means that for someone to encrypt messages for Alice, one needs to know both \(n\) and \(e\). Alice’s private key consists of \((n, d)\). This is why \(e\) is often called the encryption key, whereas \(d\) is called the decryption key, but both keys also need \(n\), which is public.

Now, to encrypt a message \(m\) for Alice, Bob would calculate the ciphertext \(c\) with the following formula: \(c = m^e \mod n\).

Alice could decrypt \(c\) to distill the original message \(m\) by using the following formula: \(m = c^d \mod n\).

As you can see from this example, the only secret part of the key is \(d\), which is required for decrypting any messages sent. But to calculate the value of \(d\), one needs to have access to \(\phi(n)\), or to \(p\) and \(q\) (because from those two, you can calculate \(d\)).

1.5 The problem with classical cryptography

The problem with all currently used algorithms, such as RSA, is that their security is based on the fact that reversing the process or brute-forcing it is “computationally hard”. This means that the sole reason they are deemed secure is that it requires very much calculation to retrieve the private key from the public key, or to retrieve the shared key from the eavesdropped communication.

In the case of RSA this lies in the fact that finding the prime factors of the public modulus \(n\) is deemed very hard since there are no known algorithms to efficiently factorize a big number.

The problem with this assumption is that, over time, computers get faster and faster, and as such also those computations do. This means that with a modern computer, it would cost (on average) over a thousand years to compute a 2048-bit private RSA key from a public key, but with a computer one can buy in two years, it may cost “just”

\textsuperscript{1}Rivest, Shamir, Adleman 1978
one hundred years. And when so-called quantum computers will be produced, those keys could perhaps be cracked in a few minutes.

This is where quantum cryptography comes in: quantum cryptography should be secure independently of this assumption. Almost paradoxically, then, quantum cryptography is a safeguard against possible future quantum computers!

So far, quantum cryptography mainly consists of Quantum Key Distribution (QKD), which exploits the laws of quantum mechanics to allow secure key exchange between two parties. In principle, quantum key distribution is unconditionally secure, so that for the security of the system it should be irrelevant to which devices and computing power any attacker has access. But here also comes the catch: in theory, quantum cryptography would guarantee perfect secrecy. But getting the devices as perfect as is needed for this purpose is very hard in practice. We have investigated whether or not they actually do guarantee this perfect secrecy, and if not: what is still wrong with them?
Chapter 2

A very brief introduction to quantum mechanics

Quantum mechanics is the revolutionary theory that arose at the beginning of the 20th century to resolve fundamental problems of the physics of really small objects. For a more detailed description one can refer to any introductory textbook on quantum mechanics, such as the book by D.J. Griffiths.[11]

2.1 States

The basic element of quantum mechanics is a state. A state can be seen as a complete description of the system under consideration. For example, when looking at a photon in the case where we are only interested in its polarization, the state is specified by the direction of polarization. The collection of all possible states of the system under consideration is called its state space.

In quantum mechanics, the state space is embedded in a so-called Hilbert space, that is, in a complex vector space equipped with an inner product. In all cases that will be looked at in this report, this vector space turns out to be finite-dimensional. In the context of quantum mechanics, vectors are usually denoted by so-called kets $|A\rangle$, where $A$ is some label. The inner product of such vectors $|A\rangle$ and $|B\rangle$ is written as $\langle A|B\rangle$. A vector $|A\rangle$ has unit length (i.e., is a unit vector), if $\langle A|A\rangle = 1$. Such vectors are very important in quantum mechanics, because states are mathematically described by unit vectors. However, it turns out that two states $|C\rangle$ and $|D\rangle$ are physically equivalent, in that they represent the same physical situation, if and only if there is a complex number $\gamma$ of absolute value $|\gamma| = 1$, such that $|C\rangle = \gamma |D\rangle$. So, more precisely, quantum-mechanical states are unit vectors (in some Hilbert space), up to equivalence in the above sense. In particular, the overall phase $\gamma$ in a unit vector $\gamma |A\rangle$ is physically irrelevant in so far as $|A\rangle$ is a quantum state.
2.2 Superposition

States may, of course, also be defined in classical mechanics (think, for example, of the position and momentum or velocity of a projectile). There is a very fundamental difference between classical and quantum-mechanical states, though. In classical physics there is no real way to combine two states into a new state (except probabilistically, as in statistical physics). In quantum mechanics, however, two states $\lvert A \rangle$ and $\lvert B \rangle$ may be combined (even in many different ways) into a new state called a superposition: this is a (complex) linear combination

$$\alpha \lvert A \rangle + \beta \lvert B \rangle,$$

where $\alpha$ and $\beta$ are complex numbers that are arbitrary, except for the fact that $\alpha \lvert A \rangle + \beta \lvert B \rangle$ again needs to be a unit vector. For example, if $\lvert A \rangle$ and $\lvert B \rangle$ are orthogonal, in the sense that $\langle A \lvert B \rangle = 0$ (and, of course, $\langle A \lvert A \rangle = \langle B \lvert B \rangle = 1$), then

$$|\alpha|^2 + |\beta|^2 = 1.$$

A simple example is $\alpha = \cos \phi$ and $\beta = \sin \phi$, for some angle $\phi$ (between zero and $2\pi$), giving a superposition

$$\cos \phi \lvert A \rangle + \sin \phi \lvert B \rangle.$$

Another example, still assuming $\langle A \lvert B \rangle = 0$, is

$$\left( \lvert A \rangle + \delta \lvert B \rangle \right) \sqrt{2},$$

where $\delta$ is a single complex number of absolute value $|\delta| = 1$. Parametrizing $\delta = \exp(i\phi) = \cos \phi + i \sin \phi$, the angle $\phi$ is then called the relative phase between $\lvert A \rangle$ and $\lvert B \rangle$. More generally, it follows from our earlier remark on the equivalence between $\lvert A \rangle$ and $\gamma \lvert A \rangle$ that, in a superposition $\alpha \lvert A \rangle + \beta \lvert B \rangle$, what matters physically is the relative phase between $\lvert A \rangle$ and $\lvert B \rangle$.

2.3 Measurements

All the above is just formalism: we also need to know how the concept of a state relates to what we can see in the real world. Here another (but closely related) difference between classical and quantum physics shows up. Classically, performing a measurement comes down to asking the system in which state it is, and getting the exact answer. If performed well, process is assumed to not alter the system under consideration.

Life is not so simple in quantum mechanics. During a measurement of a quantum state you take a collection of orthogonal states, which together span the entire state space (that is, an orthonormal base), and ask the system in which of these states it is. The result will always be exactly one of the states in that collection, and afterwards, if one measures again, one will always get the same result. This means that after the measurement, the system is in one of the states in the collection used to measure it, rather than in its original state. This remarkable fact is especially true if the system was originally in a superposition of several states in the collection defining the measurement, implying that a measurement on a quantum system disturbs that system. This fundamental property of
quantum mechanics is one of the cornerstones that make the amazing results of quantum cryptography possible.

We would also like to make predictions on the outcomes of a measurements. Remarkably, the result of a measurement is to some extent random. We can, however, calculate the probability of each possible outcome. Suppose $|A\rangle$ is the state of the system and $|B\rangle$ is an element of the collection of states used for the measurement. The probability $P(B|A)$ that we get outcome $|B\rangle$ in state $|A\rangle$ is then given, through the inner product, by

$$P(B|A) = |\langle A|B \rangle|^2.$$ 

To illustrate this formula, let us take a look at a photon, with degrees of freedom restricted to just polarization (hence forgetting its motion). This system has two orthogonal base states, which together generate the entire state space through superpositions. We have considerable freedom in choosing these base states. First, we may choose to use the base consisting of horizontal (linear) polarization ($|H\rangle$) and vertical (linear) polarization ($|V\rangle$). We can define relative angles for these by stating that the polarization angle of the horizontal state is zero by convention, angles increasing clockwise. The state of a photon with linear polarization angle $\phi$ then becomes

$$|\phi\rangle = \cos \phi |H\rangle + \sin \phi |V\rangle.$$ 

When we measure whether its polarization is horizontal ($H$) or vertical ($V$) in this state, according to the above formula the probability of each outcome is:

$$P(H|\phi) = \cos^2 \phi;$$
$$P(V|\phi) = \sin^2 \phi.$$ 

Another example is given by base states of circular polarization. Here a base is formed by states $|0\rangle'$ and $|\pi\rangle'$, which may be expressed in the previous base by

$$|0\rangle' = (|H\rangle + i|V\rangle)/\sqrt{2};$$
$$|\pi\rangle' = (|H\rangle - i|V\rangle)/\sqrt{2},$$

respectively. The state of a photon with phase angle $\varphi$ then becomes

$$|\varphi\rangle' = \cos(\varphi/2)|0\rangle' + \sin(\varphi/2)|\pi\rangle',$$

with $\phi$ between 0 and $\pi$. In that case, we obtain analogous formulae for the probabilities of the outcomes of the measurement with respect to the original set of states, like

$$P(H|\varphi') = \frac{1}{2}(\cos^2(\varphi/2) + \sin^2(\varphi/2)) = \frac{1}{2};$$
$$P(V|\varphi') = \frac{1}{2}(\cos^2(\varphi/2) + \sin^2(\varphi/2)) = \frac{1}{2}.$$ 

2.4 Combinations of systems

So far, we talked about single systems. However, in many cases we have a system that can be seen as the combination of multiple subsystems. It will be important for us to
translate the concepts introduced so far to combinations of systems. Suppose we have two systems, system 1 being in state \( |A\rangle \) and system 2 being in state \( |B\rangle \). We say the combination of the two systems is in state

\[
|A\rangle \otimes |B\rangle,
\]
called the tensor product of the two. The \( \otimes \) operator is bilinear, meaning that

\[
(|A\rangle + |A'\rangle) \otimes |B\rangle = (|A\rangle \otimes |B\rangle) + (|A'\rangle \otimes |B\rangle);
\]

\[
|A\rangle \otimes (|B\rangle + |B'\rangle) = (|A\rangle \otimes |B\rangle) + (|A\rangle \otimes |B'\rangle).
\]

The combination of the two systems can also be in states other than those that can be written as \( |A\rangle \otimes |B\rangle \). The theory also allows sums of those states, so that the system described above can also be in a state

\[
(|A_1\rangle \otimes |B_1\rangle) + (|A_2\rangle \otimes |B_2\rangle),
\]
up to normalisation (as a unit vector). In such a state it is not always possible to find two states \( |A\rangle \) and \( |B\rangle \) of the subsystems such that the system is described as \( |A\rangle \otimes |B\rangle \). When this is impossible we call the two subsystems entangled.

2.5 Transformations

The last thing we need to look at is dynamics in quantum systems. We might want to perform some operations to the systems we are working with in quantum key distribution, aside from measuring them. This is possible, but there are some limitations to the transformations we can apply in quantum theory.

First, they must be bijective, meaning that we must be able to undo the transformation, given only the state after it has been applied. Second, the transformation must be linear: if it transforms \( |A\rangle \) to \( |A'\rangle \) and \( |B\rangle \) to \( |B'\rangle \), then it transforms \( \alpha |A\rangle + \beta |B\rangle \) into \( \alpha |A'\rangle + \beta |B'\rangle \). The last condition is that it must preserve the inner product, in other words,

\[
(A|B) = \langle A'|B'\rangle.
\]

The properties of the admissible transformations are jointly called unitarity.

Unitarity leads to a very important property of quantum mechanics, called the no-cloning theorem. Roughly speaking, this theorem states that it is impossible to clone, or duplicate, unknown quantum states. More precisely, there exists no unitary transformation and no state \( |B\rangle \) of some (single) system such that, for arbitrary \( |A\rangle \), the state \( |A\rangle \otimes |B\rangle \) of the combined system consisting of two copies of the given system, is transformed to the new state \( |A\rangle \otimes |A\rangle \). Note that for some fixed state \( |A\rangle \) one can certainly find a suitable state \( |B\rangle \) and a suitable unitary transformation that accomplishes the cloning \( |A\rangle \otimes |B\rangle \mapsto |A\rangle \otimes |A\rangle \); the point is that this is impossible for arbitrary \( |A\rangle \) (which is typically unknown).
Chapter 3

Quantum key distribution protocols

In the rest of this paper we will focus on quantum key distribution. This is the secure exchange of secret keys over a public channel between two parties, potentially over long distances. First, we will discuss various protocols. This is then followed by an overview of how security proofs are structured and work.

In order to discuss its possible weaknesses we first need an understanding of how QKD works. For our research there are two important approaches to do this. One is based on the exchange of qubits. Algorithms in this class have usually been proven theoretically secure. The other approach is using so-called coherent states, and then using interference between pairs of pulses to detect eavesdropping. This approach has not yet been proven secure.

Any quantum key distribution protocol consists of two steps, which depending on the protocol and implementation are done either at the same time or one after the other. The first step consists of exchanging some random bits across a quantum channel. The second step consists of checking for errors in the exchanged sequence, correcting them and then shrinking the bit stream to compensate for all the information leaked to any potential adversaries. Here we will mainly discuss exchanging the bits across a quantum channel and the error detection, the other steps will be treated later on.

For all protocols discussed here there is a necessity for two channels. First and foremost a quantum channel between the two parties, and then a classical and authenticated channel (meaning that Bob and Alice have a way of communicating knowing that the messages they get really come from the other party).

3.1 Qubit based

One of the approaches to QKD is using so-called qubits. These are systems with a state space spanned by two orthogonal states, conventionally called \(|0\rangle\) and \(|1\rangle\). These states are normalized and orthogonal by definition. The communication is then done by exchanging a number of qubits between Alice and Bob.
Examples of physical systems which behave according to the model we have for qubits are photons, whose polarizations can be used as a qubit.

3.1.1 BB84

The first QKD protocol, based on qubits, was published by Charles Bennet and Gilles Brassard in 1984. The exchange is done by Alice preparing qubits in a specific state and Bob measuring them.

During the protocol Alice and Bob use two bases. The first is the base formed by $|0\rangle$ and $|1\rangle$. This is what we will call the computational base. The other is the so-called Hadamard base, which consists of $|+\rangle = \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle)$ and $|-\rangle = \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle)$. $|+\rangle$ will be interpreted as meaning 1 and $|-\rangle$ will be interpreted as meaning 0.

First Alice prepares $n$ qubits as follows. For each qubit she randomly chooses whether it will represent a 1 or a 0. For each qubit she then randomly chooses a base in which she will encode it. She then sends the qubits to Bob.

Bob chooses for each qubit a random base in which he will measure. He notes the outcome and base of each measurement. After he has done this he communicates with Alice over a public authenticated channel which qubits were measured in which base. Both parties then throw away all bits for which they did not use the same base.

A random subset of the remaining bits is then exchanged between Alice and Bob to determine the amount of bits eavesdropped. This works because if an eavesdropper measures in a different base than Bob does, she has a 50% chance of introducing an error in Bob’s result. This effectively means that for 25% of the bits eavesdropped Bob will get a different result from Alice, even though he measured in the same base.\cite{5}

3.1.2 Ekert 91

An improvement on the BB84 protocol was published in 1991 by Artur Ekert. Instead of using two independent bases, it uses three bases for Alice, and three bases for Bob, with only a partial overlap between the set of bases used by Alice and that used by Bob. They usually are indicated by angles, where a base of angle $\theta$ means the base consisting of $\sin(\theta)|0\rangle + \cos(\theta)|1\rangle$ and $\cos(\theta)|0\rangle - \sin(\theta)|1\rangle$. Alice then uses the bases $0^\circ$, $45^\circ$ and $90^\circ$, whereas Bob uses $45^\circ$, $90^\circ$ and $135^\circ$.

During the quantum bit exchange phase either Alice, Bob or a third party that is untrusted, produces so-called EPR-pairs. These are pairs of photons in the state $\frac{1}{\sqrt{2}} ((|0\rangle|0\rangle + |1\rangle|1\rangle))$. These have the special property that if Alice and Bob measure in the same base, then no matter what that base is, their results will always be perfectly correlated.

One member of each pair is sent to Bob and the other to Alice. Both now choose a random base to measure their qubit in. After the qubit exchange phase they exchange the bases used. The bits they measured in the same base now provide the bits for the key, whilst those that have not been measured in the same base are used to calculate the divergence from the so-called Bell inequalities, which is in turn used to calculate the number of bits eavesdropped. The advantage of this is that it is not necessary to use bits that could also have been used for the key to calculate the number of bits eavesdropped. In this way, the amount of data available for a key is potentially raised.\cite{5}
3.1.3 SARG04

SARG04 is another improvement over plain BB84. It uses the same states as used for BB84, but Alice, instead of sharing the base she used for the measurement, indicates that she prepared one of the four non-orthogonal pairs of states and Bob only says whether his measurement result was conclusive. Bob’s measurement was conclusive if it was orthogonal to one of the two states given by Alice. If this is the case, Bob knows that Alice cannot have sent that state, and since she sent one of two states, Bob knows she must have sent him the other. This leads to Alice and Bob now sharing a common peace of information, which can be used to construct a key.

The advantage given by this approach is that in practical implementations, it allows the implementation to still function over connections with higher losses than those tolerable for BB84. In practice, this means that the maximum distance over which a key exchange is possible increases.\textsuperscript{[26]}

3.2 Continuum based

The other approach to QKD uses coherent states. These are the quantum states that come out of a laser when it gives a single pulse. The energy put into the laser determines the average number of photons in such a pulse. However, the actual number of photons in the pulse when measured, is a random variable. This gives some interesting effects. Two pulses from such a laser can be made to interfere in an interferometer as long as the number of photons in each pulse is not measured, even when the average number of photons is smaller than zero, and a majority of the pulses is empty when measured. This can be used to exchange a key, whilst still having checks against eavesdropping. However, most of these schemes have not yet been proven theoretically secure.

3.2.1 Coherent one way

In the Coherent One Way (COW) protocol Alice sends a continuous stream of pulses. The stream is split into frames and in each frame the pulse can be either at the start, encoding a zero, or at the end, encoding a one. At the receiving end of the setup Bob splits the stream using a beam splitter, sending one part to a photon detector and the other to an interferometer. The photon detector registers the times at which it detects pulses. Bob then checks in which frame he saw a pulse at the first position and in which he saw one at the second position, discarding all frames in which he measured nothing.

The interferometer is set up such that the pulse at the beginning of frame \(i\) interferes with the pulse at the beginning of frame \(i - 1\). It is configured such that when the values represented by the frames are equal (both zero or both one), there will not be a click in the detector attached to the interferometer. If an eavesdropper measured some of the pulses, some percentage of the time she will measure nothing in a frame. In this case she can only guess at what Alice originally sent to Bob and no matter what she sends on, the result will be either an error in the sequence of bits registered by Bob, or a click in the interferometer where there should not have been one. Bob and Alice exchange some of the measurements of the interferometer and photon detector and use this to figure out how many errors there are in the stream and how many bits have been eavesdropped.\textsuperscript{[30]}
Chapter 4

Post-processing

After exchanging a string of bits via the quantum channel through one of the protocols described in chapter 3, there needs to be done some post-processing. The combination of noise and possible eavesdropping has created errors in the exchanged string and leaked information about it to an eavesdropper. When the amount of information lost this way is not too great, there are algorithms to correct for their effects.

4.1 Error correction

The first problem is that, because of transmission errors, the strings posessed by Alice and Bob might not be equal. This is corrected for during a process aptly called error correction. During this process Alice and Bob exchange bits of information about their respective views of the key and correct them based on the differences between this information. However, as all classical communications are supposed to be readable for any eavesdropper, they want to do this whilst revealing a minimal amount of information about the key.

This requirement excludes the exchange of the entire secret key over the channel. So instead, Alice and Bob calculate so-called check-bits over substrings of their respective key strings. By comparing these they can decide where corrections must be made, whilst not revealing the entire key to an adversary.

For more information, see reference [3].

4.2 Privacy amplification

Both the errors caused during the quantum communication step and the information exchange for error correction have leaked information on the key to an adversary. This information can be rendered useless with high probability in a process called privacy amplification.

The idea is that both Alice and Bob apply a transformation on their key string, producing a new, shorter string of bits independent (with high probability) of the information
which the eavesdropper has on the key. However, Alice and Bob do not know what information the eavesdropper actually has about their shared key. So, the possibility exists that the information Eve has is exactly the result of the transformation done by Alice and Bob.

This is only problematic if an eavesdropper knows up front which transformation is going to be used, because she can then use this information to carefully only extract the information she needs. This can be circumvented by Alice and Bob selecting a random transformation after the error correction phase. It can then be shown that an eavesdropper has significant information on the key only with a probability that can be made arbitrarily small.\cite{6, 4}
Chapter 5

Proving security

We will now present an outline of the important ideas behind the security proofs for quantum key distribution protocols. This will be limited to qubit based protocols, and will not be very detailed. It will, however, be suitable for identifying the assumptions in the model, which are needed later on in understanding how practical implementations differ from this model and how this allows attacks. For a more formal proof of security we refer to [6] and [25].

5.1 Model

First of all, we need a model for the quantum communications during a key exchange. We will model them as follows. Alice sends a quantum state of \( n \) qubits to the eavesdropper, whom from now on we will call Eve. Eve, having \( n \) qubits of her own, then applies a transformation to the combined systems of the \( n \) qubits and her own \( m \) qubits. Eve then sends the resulting \( n \) qubits on to Bob, who will apply a measurement on the \( n \) qubits. Eve is allowed to wait with measuring her private state until the entire protocol between Alice and Bob is finished. In this model we assume that Alice produces perfect qubits, in other words, she produces a system that is really a qubit and not an approximation. We also assume that Bob’s measurement device is perfect, such that it is not possible to make it measure something other than a qubit and that it does not leak information.

5.2 Security of QKD in the model

The no-cloning theorem now states that eve cannot make an exact replica from the \( n \) qubits she got from Alice in her \( n \) private qubits. This suggests that, when she extracts \( k \) bits worth of information from the state received from Alice, she must also leak \( k \) bits of information on her initial internal state. Since these cannot be correlated, this must introduce some amount of errors, which will be proportional to the number of bits of information she extracted from the communicated state.

Since Alice and Bob can estimate the amount of errors based on the measurements done by Bob, they can also make an estimation of the amount of information gained by the eavesdropper on their shared key. They can then use this information during error correction and privacy amplification to get a secure shared key.
Chapter 6

Commercially available products

In our search for companies that produce systems that implement quantum cryptographic applications we found only two firms. These companies include ID Quantique and MagiqTech. Of these, ID Quantique is the company we visited and MagiqTech has facilities in Boston and New-York. It is hardly surprising that there are only a few providers, since the market for quantum cryptographic applications, such as QKD equipment, is very small. The most important group of customers consists of universities, which buy these products mainly to do research on. Furthermore, we came to know that the government of Switzerland had bought a system from ID Quantique, which is used for elections. In what follows, this product will be discussed.\cite{29}

6.1 ID Quantique SA

To see the devices that are commercially available at the moment, we visited ID Quantique. This company is situated in Geneva, Switzerland. The company is a commercial spin-off of GAP-Optique, the Group of Applied Physics of the University of Geneva. The company originated about twenty years ago and ten years ago it doubled its personnel. The research at ID Quantique is done in cooperation with GAP-Optique. Of the practical part, only the final step in the production process, namely the combination of the various parts, is actually done at ID Quantique. Also testing of the final systems is done at the company.\cite{29}

In fact, the main products sold by ID Quantique are not key distribution systems, but quantum random number generators. Indeed, for this type of applications the market is a lot bigger. The device they sell is called Quantis. It exploits an elementary quantum optics process. The randomness depends on a semi-transparent mirror. Photons are sent one by one onto this mirror, and whether the photon is reflected or let through shows the value of the bits, zero or one. Being deterministic, computers are not really capable of generating random numbers. However, random numbers are required for many applications. So a quantum random number generator is a useful device. For instance,
they are used by casinos on a large scale.

The company also sells scientific instrumentation, notably single photon detectors and lasers. These products are also necessary components in the quantum cryptographic key distribution systems. We now come to the most important device for our research, the QKD system, which is called Clavis\textsuperscript{2}. (The full name is id3100/id3110 Clavis\textsuperscript{2} Quantum Key Distribution System.) Actually, this is the only QKD device that is commercially available. It has its name due to the Latin meaning of the word “clavis”, which is “key”.

This system costs about 100,000 euros and reflects a growing interest in the scientific community, so this product is the one on which ID Quantique puts its chances. This is also the system actually in use at the Swiss government. According to the brochure of ID Quantique, the system makes it possible to securely exchange keys over distances of tens of kilometers. And, also according to their brochure, “it is the most flexible product of its kind on the market”.

Figure 6.1: The boxes Alice and Bob of the Clavis\textsuperscript{2} system of ID Quantique.

One Clavis\textsuperscript{2} system consists of two stations or two boxes, which can be seen in figure 6.1. As usual, one of the boxes is called Alice (QKDS-A) and the other one is called Bob (QKDS-B). One of the two is an id3100 Clavis\textsuperscript{2} and the other one is an id3110 Clavis\textsuperscript{2} system. The difference between the two numbers concerns the length of the delay line. The id3100 station has a delay line of twelve kilometers, whereas id3110 has a delay line of 24 kilometers. The boxes are controlled by an external computer, which must run on a the Ubuntu version of the Linux operating system.

6.2 Specifications of Clavis\textsuperscript{2}
There are two possible configurations of the system, a single- or double-computer configuration. In the single-computer configuration, one computer is used to control both the QKDS-A and QKDS-B stations. To control both boxes, two instances of the control programme must be ran on the same computer. Because there is just one computer that shows the key, this configuration does not allow key exchange over a distance to different locations. This configuration, though, is useful for testing and calibration purposes. Only the set-up with two computers can be used for key exchange to two different locations. In this configuration each device is controlled by a different computer, whilst a management (classical) channel is required for synchronization of the system and key distillation. This channel is implemented over a TCP/IP connection in a local area network, like the internet or a dedicated optical fibre. The computers can be linked to external encryption devices or other applications, for which the computers can provide the cryptographic key material.

### 6.2.1 Software

The process of key distillation is called post-processing, which is fully implemented and automated in the systems. Post-processing of the raw key material consists of four main steps:

1. Sifting implemented both for BB84 and SARG04
2. Distillation of the key which consists of two steps:
   - Key reconciliation, which is performed by the Cascade algorithm\[^3\]
   - Privacy amplification, which is performed by Wegman-Carter Strongly Universal Hashing\[^4\]
3. Authentication, which is performed by universal-hashing with One-Time Pad encryption\[^4\]
4. Key material storage and management

The software suite consists of five different applications. First, the QKD Menu Application, second a QKD Sequence Application, third a QKD Log Analyzer Tool, fourth a QKD Device Access Library and finally a QKD Secure Chat Tool. A short explanation of these applications is as follows. First, the QKD Menu Application is a tool by which the id3100/id3110 Clavis\(^2\) system can be controlled and operated. Access to all the hardware parameters is provided by this application. Second, the QKD Sequence Application is a fully automated programme that controls the Clavis\(^2\) system. It sequentially performs the tasks required for quantum key distribution. This application also stores the cryptographic key material that is produced in a key storage. This key storage can be accessed by other applications and external devices. Third, the QKD Log Analyzer Tool is a programme by which the log files produced by Clavis\(^2\) can be read. It can also be used to graphically display key parameters and variables in order to analyze their temporal evolution. Fourth, the QKD Device Access Library is a library of more than fifty functions that can be used to programme the system. It allows users to write customized programmes accessing the system to perform the tasks required by quantum key distribution. Finally,
the QKD Secure Chat Tool is a messaging application allowing the exchange of encrypted messages between computers connected to the stations of the Clavis\(^2\) system. For more specifications of the Clavis\(^2\) system see the brochure of ID Quantique.\(^{(22)}\)

Now that we have seen the whole software suite of the system, implementing the quantum key distillation, we shall take a look at the physical implementation of the system.

### 6.2.2 Physical implementation

![Figure 6.2: Quantum key distribution between Alice and Bob with Clavis\(^2\).](image)

The schemes of both the devices are shown in appendix 10 (page 41). We will now explain the devices on the base of these schemes. The main text will not explain all of the components, for a detailed explanation of all the components see appendix 10 (page 43). Furthermore, the laser and the detector are explained in section 7.3. The id3100/id3110 Clavis\(^2\) system is based on an auto-compensating interferometric set-up. Hereafter, we will explain what this means.

Roughly speaking, what happens inside the device is that a light pulse is sent from Bob to Alice. Alice attenuates the light beam to a single-photon-level amplitude and modulates the relative phase of the photon. A Faraday mirror reflects the photon back to Bob. The photon that goes back to Bob is detected in one of the two detectors. This set-up is called auto-compensating or, the so-called plug-and-play design. This QKD system is based on a interferometer. A schematic illustration is shown in figure 6.3.

There are a couple things that have to be kept in mind before we go through the system process. First, the measurements are based on the relative phase of the photons, so this phase carries the quantum information. There are two measurement bases, 0 and \(\pi\). In the first base the phases 0 and \(\pi\) represent the bits 0 and 1. In the second base, \(\pi\)
Figure 6.3: The interferometric set-up of Clavis\textsuperscript{2}. Alice can choose between the four states: 0, $\frac{\pi}{2}$, $\pi$ en $\frac{3\pi}{2}$. Bob can choose between the two bases, 0 and $\pi$.

and $\frac{3\pi}{2}$ represent the bits 0 and 1. Second, the polarization of the photons is only used to make sure that all photons travel over the same distance, rather than for any quantum state measurements. Third, the laser sends a beam of photons. However, it is intended that just one photon eventually reaches Alice. This is the reason why the light intensity is reduced during the process. It is reduced in such a way that from every pulse of light eventually on average one third of a photon remains.\textsuperscript{29}

Figure 6.2 gives a view on the inside of the boxes of Clavis\textsuperscript{2}. We will now explain, step by step, the path of the photons through these boxes. First, Bob’s laser sends a beam of photons, which is then separated by the 50/50 coupler after passing the circulator. The phase modulator, which the photons then pass, has no effect on this light beam yet. Then the photons pass the polarizing beam splitter, which splits the photons in two beams according to the polarization of the photons. Then, one of these two light beams is lost. The upper light beam in the scheme is then split by a 10/90 coupler to reduce the amount of photons in the direction of the Faraday mirror. The bottom light beam is attenuated and the photons are measured by three different detectors. These detectors are used for calibration processes and also to verify the light intensity en frequency to perform a first check on whether there is an eavesdropper active.

The upper light beam is also attenuated and subsequently the bandpass filter only selects photons of a frequency of about 200 Gigahertz. Then the delay line and the phase modulator, which again has no effect yet, are passed. At this time, there is only one photon left, which is in a superposition. This means that this one photon took both paths at the beginning, so it kind of “splits”. Then the photon is reflected with
orthogonal polarization, by the Faraday mirror. A little further, it becomes clear why
this is important. On its way back, both “superpositions” of the photon first pass Alice’s
phase modulator. Alice then chooses a phase shift for this photon, which is done by a
random number generator. She can choose the rotations $0$, $\frac{\pi}{2}$, $\pi$, and $\frac{3\pi}{2}$, but she only
rotates the phase of one of the two “superpositions”, the other one is the reference signal.
The phase difference that is generated is then the quantum state of the photon.

Then the photon returns its way back to Bob and passes the polarizing beam splitter
again. Because of the orthogonal polarization, the photon will now take the other path
than it did before. However, the photon took both paths the first time, so the “super-
positions” will take both paths again, but conversely, relative to the first time. Now the
reference signal takes the upper path and passes the second phase modulator, which is
Bob’s. Bob now chooses the base in which he will measure. The random number gener-
ator decides if this base will be 0 or $\frac{\pi}{2}$, which causes the phase of the reference signal to
change. Then both “superpositions” reach the 50/50 coupler exactly at the same time,
because they both traveled over the same distance. At this point interference between
both “superpositions” takes place. Due to particular properties of the 50/50 coupler,
it can translate the quantum information, which is encoded in the relative phase of the
“superpositions”, into the classical bits 0 and 1. When there is no phase difference, the
photon will be transmitted and detected. On the other hand, when there is a phase
difference of $\pi$, the photon will be reflected and detected by the other detector. These
two possibilities represent the classical bits 0 and 1. In case the base chosen by Bob is
not equivalent to that chosen by Alice, the phase difference will be $\frac{\pi}{2}$, and whether the
photon will be reflected or transmitted is completely random. This determines that only
the qubits measured by Bob, in the same base that Alice chose, are useful for the eventual
key.\textsuperscript{[29]}
Chapter 7

Weaknesses of QKD devices

As we already mentioned, QKD protocols guarantee perfect security based on the laws of quantum physics. However, security proofs of these protocols necessarily assume an idealized model of equipment. Unfortunately, there is a big gap between this idealized model and currently commercially available quantum cryptographic devices. Loopholes caused by the discrepancies between the model and the actual devices can be used by eavesdroppers. Challenges of the physical implementations of QKD protocols can roughly be divided into three categories. There can be problems at both the preparation and the measurement. Then, the source and the detector must also be connected in some way. In this chapter, we will describe these technological challenges. In the next chapter we will discuss various possible attacks on quantum cryptographic devices, which utilize weaknesses in the practical implementation.

7.1 Preparation

In the protocols BB84 and SARG04 (see also sections 3.1.1 and 3.1.3), Alice is assumed to use a single photon source for the preparation of her states. But perfect single photon sources do not exist in reality. Therefore, in practical implementations faint laser pulses are used to approximate single photon number states. In these faint laser pulses the photon number distribution obeys Poisson statistics. Of course, there is the small probability of generating more than one photon at the same time when using such a source. This multi-photon component is the main problem with these practical sources, since even small fractions of multi-photons can have big consequences for the security of the key for large losses in the quantum channel.\textsuperscript{[9]} The security loophole caused by these multi-photons is exploited in the photon-number-splitting attack, which we will discuss later.\textsuperscript{[28]}

Since the probability of finding $n$ photons in a coherent state produced by a practical source follows Poisson statistics:

$$P(n, \mu) = \frac{\mu^n}{n!} e^{-\mu},$$

(7.1)
the probability that a nonempty weak coherent pulse contains more than one photon is

$$P(n > 1 | n > 0, \mu) = \frac{1 - P(0, \mu) - P(1, \mu)}{1 - P(0, \mu)} = \frac{1 - e^{-\mu}(1 + \mu)}{1 - e^{-\mu}} \approx \frac{\mu}{2}. \quad (7.2)$$

Hence, this probability can be made arbitrarily small by decreasing the mean photon number $\mu$. However, when $\mu$ is really small, which is necessary for minimizing the effects of multi-photon states, most pulses are empty, for

$$P(n = 0) \approx 1 - \mu.$$ 

It is possible to compensate for the decrease in bit rate caused by these empty pulses, thanks to high modulation rates of the lasers. But a high modulation rate results in more dark counts. A dark count means that a detector gives a click, even though no photon is arriving. Therefore, really low mean photon numbers cannot be used. In most experiments the value $\mu = 0.1$ is used,[1] by which approximately 5% of the nonempty pulses contains more than one photon. There exists an optimal value for $\mu$, depending on the transmission losses.[16] So, despite of these high modulation rates, using faint laser pulses still results in a relatively low speed of QKD.

Of course, an ideal single photon source, which emits exactly one photon whenever Alice wants it to, would solve the multi-photon problems. Due to their low quantum efficiencies, these sources, so-called photon guns, currently do not offer an advantage over faint laser pulses with a very low mean photon number.[9] However, in the future there might be some improvements in this field.

7.2 Channels

When Alice and Bob want to use QKD, they need to connect the source and the detector by a communication channel that can transmit quantum information, a so-called quantum channel. Since in practical implementations photons are used as qubits, light must be transmitted from Alice to Bob in some way. This could be accomplished either through an optical fibre or through free space.

Free-space QKD can be used when no optical fibre is available or in satellite communication, but since this is not yet commercially available, it is beyond the scope of our text and we will only discuss it briefly here. However, we will mention the possible future for free-space quantum cryptography in the discussion at the end of this review. Since with free-space QKD the signals are not transmitted in a fibre, the transmitted energy spreads out, which leads to higher transmission losses. Moreover, light from outside can lead to a higher error rate. Even at night moonlight and city light could cause errors. Also the weather could influence free-space QKD. Finally, the performance of free-space systems depends highly on atmospheric conditions and is not possible in for example fog.[9, 23]

QKD through optical fibre is not influenced by the weather or by diffused light, but it does have its own difficulties. First of all, Alice has to avoid any correlation between the polarization of the photons and their frequency, for an eavesdropper could use such a correlation to gain information about the polarization of the qubits. Another challenge is that the bases that Alice and Bob use must be related by a stable transformation. For instance, the systems can be aligned by defining the vertical and diagonal directions.

28
When Alice and Bob know this transformation, they can compensate for the difference in their bases. In this way, they can obtain the required correlation between preparation and measurement. A big problem with quantum channels is that the transmission losses depend exponentially on the length of the fibre.\textsuperscript{[27]} And since lost photons are correlated to the signal, they must be counted as information gained by Eve. Therefore, the length of the optical fibre used for QKD is bounded, which means Alice and Bob cannot be too far apart.\textsuperscript{[27]}

7.3 Detection

The last technological challenge we discuss here consists of the problems at the level of detection. The detector is the Achilles’ heel of most practical implementation of QKD, as most possible attacks aim at the detectors. An ideal single photon detector must have a high detection efficiency for a large range of frequencies. Also, the probability of giving a signal without a photon arriving (a dark count) must be as small as possible. Besides, the dead time of the detector, which is the time it takes to reset the detector after a click, must be short, since to date high data rates are only possible if the detector recovers quickly. Unfortunately, no real single photon detector completely meets these conditions. We will discuss two kinds of single photon detectors and their possible weaknesses here: threshold detectors with avalanche photodiodes and superconducting single photon detectors.\textsuperscript{[28]}

7.3.1 Avalanche photo-diodes

An avalanche photo diode (APD) detects single photons by measuring the breakdown of a photo diode. A voltage is applied over the photo diode opposite of the direction of conductivity of the diode. It is slightly beneath the breakdown voltage, such that a single photon can trigger the breakdown of the diode. When a breakdown happens the diode rapidly starts to conduct current, which in turn leads to a decrease in voltage over the diode. This drop in voltage is then measured and used as the output of the avalanche photo-diode.

The drop in voltage also causes the diode to stop conducting, and the voltage can then slowly rise to slightly above the breakdown voltage, after which the detector is again ready to detect the next photon. During drop and rise of voltage the detector can effectively not measure another photon when it arrives, which limits the amount of photons the detector can handle.

These detectors typically operate at temperatures relatively close to room temperature, which makes them easier to handle, but is also the reason for their significant amount of dark counts.

7.3.2 Superconducting nanowire single photon detectors

Superconducting nanowire single photon detectors (SNSPD) consists of a thin wire which is cooled to superconducting temperatures. A current is then run through it, close to its critical current (the current at which it stops superconducting). When a photon then hits the wire, it will heat it slightly at its point of impact, locally reducing the critical current, which causes the wire to cease being a superconductor. This jump in resistance
is measurable and causes the click. The rise in resistance also reduces the current through the wire, causing it to cool down and become superconductive again.

SNSPD's have the advantage that, because of their low temperature, their dark counts are rather low. Also, the process of cooling the wire is relatively fast, resulting in higher count rates being achievable.

7.3.3 Faked states

A problem with detectors is that faked states can be generated. Faked states are light pulses generated by Eve, which can force a detector to produce outcomes resembling those corresponding to the detection of single photons. Faked states can be used for intercept-resend attacks that we will discuss further in this text. Another difficulty is that photon detectors can be manipulated so as to change their timing behavior. In this way, Eve can make the detection time partly correlated with the detection outcome. This can be used for time-shift attacks, which we will also explain later. Another attack that aims at the photon detectors is called quantum blinding.
Chapter 8

Possible attacks

Since in practical implementations of QKD the devices are not perfect, Eve can do a hacking attack, which is related to the weakness of a specific implementation. Here we will discuss several possible attacks: intercept-resend attacks, the photon-number-splitting (PNS) attack, the time-shift attack and quantum blinding attacks.

8.1 Intercept-resend attacks

The simplest type of possible attack is the intercept-resend attack. Eve intercepts the photons sent by Alice and measures them in randomly chosen bases. Then she sends replacement states to Bob, prepared in the base she used for measurement. In QKD this will lead to errors in the key. Eve has a 50% probability to measure the photons in the right base, and when she uses the wrong base, she gets a random result which still has a 50% probability to be the right outcome. Therefore, the probability an intercepted photon generates an error in the key string is $50\% \cdot 50\% = 25\%$. When Alice and Bob are done transmitting the photons, she and Bob compare $n$ of their key bits, using a public channel. If Eve has eavesdropped all bits, the probability they find disagreement and thus identify the presence of Eve, is $P_d = 1 - (\frac{3}{4})^n$. Therefore, if Alice and Bob compare $n = 72$ key bits, there is a probability $P_d = 0.999999999$ to detect Eve. So thanks to the laws of quantum physics, this attack does not really work well theoretically, since it is easy for Alice and Bob to detect an eavesdropper with high probability.

8.1.1 Faked states attack

But, fortunately for Eve, there are more possibilities for intercept-resend attacks due to weaknesses in the practical implementation of QKD systems. An example of such an attack is the faked states attack, which we will discuss now. In a faked states attack on a QKD system, Eve does not try to reconstruct the original states sent by Alice, but generates light pulses, called faked states, that are detected by Bob in a way controlled by Eve. A successful faked states attack would give Eve full knowledge of the key. Faked states attacks can be run against several systems, even against systems with active base choice on Bob’s side. In these attacks Eve forces Bob to measure the states in a base
chosen by Eve, instead of a randomly chosen base. Eve breaks into the channel between Alice and Bob and connects the fibre running from Alice to an equivalent of Bob’s set-up. She detects every state sent by Alice and notes the detection base she uses and the bit value she measured. After that, she sends a faked state towards Bob for every state she detected, using her own detection base to send the faked state. Bob detects these states in the base Eve forces him to. Alice and Bob will not notice Eve’s presence, because after the key sifting process all the key bits have been detected by Eve in the proper base. Therefore, when Alice and Bob compare their key bits, they will not find an increase in the quantum bit error rate. Eve can then watch the classical public communication between Alice and Bob and perform the same sifting, error correction and privacy amplification as Bob to obtain the same key.\[8, 20\]

The faked states attack contains a few challenges for Eve. First of all, she has to break into the channel between Alice and Bob. Eve needs two switches for this, one for splitting off photons from the line and another one for sending pulses down the line to Bob. The easiest possibility for this would be installing a tap before installation of the quantum link. If Eve knows about installation plans in advance, she could install the equipment onto the fibre and wait until Alice and Bob begin to use it for QKD. If this is not the case, she could install a tap when the channel is not in operation, for instance during maintenance. If the channel is constantly running, Eve would need a method that does not interrupt the channel. This is of course the most difficult option for her. When Eve has broken into the channel, she still needs a way to control the bases in which Bob measures. Eve can do this by making use of the weaknesses of Bob’s detector. There are several known ways to force the desired response from Bob’s device. For instance, base choice via polarization,\[20\] time-shift attacks and quantum blinding. We will discuss the last two kinds of attacks later in this review. Faked states attacks are difficult for Eve and most of the time they are not very likely to succeed. But the fact that it is hard for Eve does not mean it guarantees perfect secrecy for the quantum cryptosystem.

8.1.2 Phase-remapping attack

Another intercept-resend attack is the phase-remapping attack. This is a so-called Trojan horse attack, which is an attack in which Eve sends signals that enter Alice’s and Bob’s devices through the quantum channel. One key assumption of QKD protocols is that Alice can prepare the required quantum states without errors. Even though this might seem like a simple assumption, it may be violated in practical QKD systems. Phase-remapping attacks exploit this loophole.\[33\]

In a plug-and-play QKD system, like the ID Quantique system discussed in chapter 6, Bob sends two laser pulses to Alice, a signal pulse and a reference pulse. The actual information is encoded in the relative phase between both pulses. Alice uses the reference pulse to determine when she should activate her phase modulator. Then Alice only modulates the phase of the signal pulse, attenuates the pulses to single photons and sends them to Bob. Since Alice allows signals to go in and out of her device to do this QKD, Eve gets the chance to do the phase-remapping attack. Eve can time-shift the single pulse so that is arrives at the phase modulator earlier or later than it should. Then the signal will be partially modulated and the relative phase between the signal pulse and the reference pulse will be smaller than it is supposed to be. In this way, Eve can even
remap the phase information from \( \{0, \pi/2, \pi, 3\pi/2\} \) to \( \{0, \delta, 2\delta, 3\delta\} \).\(^7\)

Since Eve can change the phase difference \( \delta \) between the states sent by Alice, she can launch an intercept-resend attack that is optimized for this particular phase difference. If she used a simple intercept-resend attack as described earlier without phase-remapping, it would result in a quantum bit error rate of 25\%. Bob and Alice would then discard their key and start over. So, Eve needs an attack that leads to situations in which Alice and Bob believe that they can generate a secret key. It has been shown that it is possible to get a quantum bit error rate of 19.7\% (this might even be improved in the future),\(^{33}\) while an error of 20\% is tolerable in the BB84 protocol without any Trojan Horse attacks (and 19.9\% in the SARG04 protocol),\(^7\) meaning that a secret key can be distilled. If Alice and Bob are not aware of phase-remapping attacks, they would distill a key that they assume to be secret and Eve will get information about the key. Consequently, Alice and Bob will need a lower tolerable error percentage for secret key distillation.

### 8.2 Photon number splitting attack

As we already mentioned in the paragraph about photon sources, perfect single photon sources do not exist (yet). And since the photon number distribution obeys Poisson statistics, there is a small probability of generating more than one photon at the same time. When this happens, Eve can split off one of the photons and transmit the remaining photon to Bob. Eve can store the obtained photons until Bob detects the matching photons and Alice reveals the bases she used. Eve can then measure her photons in the same base and in this way obtain information on the key, without being noticed by Alice and Bob. This is called the photon number splitting (PNS) attack. Even with the possibility of a PNS attack, Alice and Bob can still generate a secure key. However, this significantly reduces the secure key rate.\(^{10}\)

A solution to PNS attacks is the use of decoy states. Alice prepares a set of states in addition to the standard states she wants to send to Bob, called decoy states. These states are only used for the purpose of detecting eavesdropping, rather than for key generation. For decoy states, Alice randomly uses different low mean photon numbers for her photon source. For example, she could send the first pulse with \( \mu = 0.1 \), the second pulse with \( \mu = 0.4 \), the third pulse with \( \mu = 0.05 \) et cetera. Each different mean photon number has its own probability of producing more than one photon in a pulse. These mean photon numbers are the only difference between the decoy states and the standard states. Therefore, Eve cannot distinguish a decoy state from a signal state and the only information she gets is the number of photons in a signal. Thus, decoy states can be implemented to secure against PNS attacks, while allowing high key rates.\(^{15}\)

### 8.3 Time-shift attack

Another attack that aims at the detection stage, is the time-shift attack. It makes use of the detection efficiency loophole. Most QKD systems have two or more single photon detectors. In an ideal situation, those detectors are identical. But in practice it is hard to build two identical detectors. Therefore, an efficiency mismatch could exist in the time domain.\(^{13}\) When a signal is sent from Alice to Bob, one of Bob’s detectors should give
a click. To minimize the dark count rate, APD’s usually work in “gated mode”: they are activated by a gate signal for a narrow window only when a signal is expected to arrive. Outside that window, the detectors do not respond to the incoming signals. At the moment the signal arrives, the efficiency of the detector should be as high as possible, to detect the signal with a high probability. Moreover, every detector should have a high efficiency at the same time. Then both detectors have the same probability to give a click. In practical implementations however, this is not always the case. When Alice and Bob calibrate their devices, such a mismatch is sometimes observed. There is a shifting between the efficiency curves. (see fig. 8.1) This mismatch can be used for time-shift attacks.

Eve can delay or advance pulses from Alice to Bob by using high speed optical switches to re-route Alice’s signal through either a long or short optical path. When there is an efficiency mismatch in the time domain, one of the detectors will have a higher probability to click when the signal is time-shifted back- or forward. But to use this, Eve needs to know in which direction the shifting is. She wants the shift to be large enough to perfectly distinguish the detectors. This is a limitation of the time-shift attack, since the mismatch is only large enough in merely 4% of all attempts. Now Eve can randomly time-shift signals back or forward and with a higher probability she knows which detector will click. But, with only a time-shift attack, Eve will not be able to get the full information on the sifted key. Then it might be still possible for Alice and Bob to create a secret key. To avoid this, Eve can use a faked states attack. Compared to the intercepted measurements, she prepares the opposite bit value in the opposite base and sends it. She can send it with such a timing that the detection of the opposite bit value is suppressed, because of the low detection efficiency. In this way, she acquires information on the key without Alice and Bob noticing.

Removing the efficiency mismatch in APD’s is hard. However, Bob can make sure
the mismatch is not too high by measuring other parameters. Another way to counter a
time-shift attack is randomizing Bob’s phase.[13]

8.4 Quantum blinding

It is possible to blind and control the detectors of quantum cryptographic devices, like
the one made by ID Quantique. Using quantum blinding, Eve can eavesdrop the full
secret key without increasing the quantum bit error rate.[17, 19] In this review, we will
discuss two quantum blinding attacks: continuous-wave blinding and thermal blinding.
Both attacks aim at APD’s, the detectors used in commercially available QKD systems.
These attacks have been demonstrated by Lars Lydersen et al.[17] The APD’s are tricked
to exit the single-photon sensitive Geiger mode, which is called blinding.[18]

8.4.1 Continuous-wave blinding

As we already mentioned, APD’s are operated in Geiger mode to detect single pho-
tons. However, part of the time the APD’s used in QKD systems are biased under the
breakdown voltage, in the linear mode. When a detector is in the linear mode, it remains
sensitive to bright light with a classical power threshold. This threshold should be defined
well enough to make perfect eavesdropping possible.[17] With continuous-wave blinding,
Eve makes the bias voltage drop to a level such that the APD is never in Geiger mode,
which makes the detector a classical photo diode. Eve can now use an intercept-resend
attack, like the faked states attack which we have discussed earlier, to eavesdrop on the
key. She detects qubits from Alice in a random base and resends her detection results to
Bob. But she uses bright pulses instead of single photons, just above the classical power
threshold. Then Bob measures the received pulses in a randomly chosen base. When his
base choice coincides with Eve’s base choice, the current caused by Eve’s pulse crosses
the threshold current. In that case the detector will give a click. But when Eve and
Bob have selected opposite bases, half the intensity of Eve’s pulse hits each of the two
detectors, since half of the photons will go through the beam splitter and half of the
photons will be reflected. The currents are now below the threshold and therefore none
of the detectors will give a click. (see fig. 8.2)[17] So, when Bob’s detectors have been
blinded, a faked states attack will not cause an increase in the quantum bit error rate.
And since the currently commercially available QKD systems operate according to the
plug-and-play principle, it is easy to install a plug-and-play eavesdropper.[17] An answer
to this kind of quantum blinding attack has been proposed by Yuan et al.[34] Removing
the bias voltage impedance or lowering the threshold in the detector would make these
attacks impossible. A new blinding attack, the so-called thermal blinding, could be used
against APD’s regardless of the impedance of the bias voltage supply.

8.4.2 Thermal blinding

It turns out that quantum blinding is also possible on low-impedance biased detectors.
This type of quantum blinding is called thermal blinding. When an APD is illuminated,
the APD temperature increases. The breakdown voltage of APD’s is temperature depen-
dent, in that it increases when the temperature increases.[18] And since the bias voltage
is constant, an increase in temperature will thus make the APD exit the Geiger mode. When the illumination is increased even when the cooling system is running at its maximum capacity, the APD temperature starts to increase approximately linearly with the heat scattered by the APD. The probability to click then decreases till it is zero. The detector is then fully blinded. In the experiment done by Lars Lydersen et al. the blinding caused clicks for several minutes before the APD’s were heated. However, when the blinding once has been turned on, Eve can remain undetected. Concluding, the detectors in commercially available QKD systems have proved to be blindable, even with a low-impedance bias voltage supply. And when blinded, the detectors can always be controlled. This allows Eve to gain full information on the secret key without Alice and Bob noticing. At this moment there is no generic secure detector available. However, it is possible to prevent blinding attacks with specific parameters. Companies like ID Quantique implement such countermeasures to improve the security of their systems.
Chapter 9

Summary and conclusion

We will try to summarize the attacks and the response of ID Quantique to them, in order to be able to derive a verdict on the security of commercially available QKD systems like Clavis\textsuperscript{2}.

First, we mentioned the intercept-resend attacks, where the eavesdropper essentially intercepts the quantum states sent by Alice, measures it in a random base, and sends a new photon to Bob using the same state as the one in which she measured. However, as explained, this is almost impossible without detection, as this will introduce errors in approximately 25% of the bits because of the choice in base she has to make. This means Eve has to perform smarter attacks if she wishes to make the chance of discovery as small as possible.

One of the smarter ways to do this is the faked states attack, where Eve forces Bob to measure in the same base as she has measured Alice’s state in. The hard part for the eavesdropper in this attack is forcing Bob to use the same base she used. This means that this attack will become impossible if the detectors of Bob cannot be forced into a specific base by an eavesdropper, or if they can detect the attack such that they can abort the protocol. This risk of attack has been minimized in the Clavis\textsuperscript{2} by watching parameters of the APD’s for any anomalies.

Another known way of performing an intercept-resend attack smartly is the phase-remapping attack. This sort of attack can be mitigated by setting a lower threshold of allowed quantum bit errors. This is a software-based option in the Clavis\textsuperscript{2}, which can be tweaked to get the level of security set by the user (in such a way as to optimize the signal/noise ratio).

Another attack we mentioned is the time-shift attack. Here, the eavesdropper takes advantage of the fact that the photon detectors used in the devices are never exactly equal to each other: the timing of when they are at their most efficient moment differs a little bit. This could be prevented if the detectors used were exactly equal in terms of their efficiency timings, which is practically impossible, or minimized by randomizing the phase used by Bob. In the Clavis\textsuperscript{2}, ID Quantique has tried to make these timings coincide as exactly as possible, but the company has not yet succeeded in solving this problem completely.

The last attack type we mentioned was the quantum blinding attack, where the eaves-
dropper blinds the detectors of Bob such that she can know for sure which detector is triggered.

The first way of blinding the detectors is the continuous-wave blinding attack. In the Clavis\(^2\), this has been prevented by lowering the threshold of the detectors.

The other quantum blinding attack is the thermal blinding attack. This is prevented in the Clavis\(^2\) system by setting thresholds for the allowed temperature of the system, such that it can detect this type of attack, and abort the protocol.

**Executive conclusion**

To conclude, we would like to point out that although quantum cryptography is theoretically unconditionally secure (in the sense that it does not matter to what sort of devices Eve has access to), the practical implementations have flaws which undermine this security. As we have pointed out in the previous chapter, the Clavis\(^2\) system is still vulnerable to the faked states and phase-remapping attacks, although the effects have been minimized. The Clavis\(^2\) is also vulnerable to the time-shift attack. This attack is very hard to fix with software, as it is based on problems with the detectors themselves, but ID Quantique will certainly keep trying to perfect their APD’s. Finally, we mentioned the quantum blinding attacks, which have been rendered almost impossible by watching the parameters of the devices very carefully, and determining normal values at the start of the protocol.

It will never be possible to make such devices completely secure as this is impossible to prove, and attackers will always find some flaws in the devices. But although perfect security cannot and has not been reached, this is also not required: the only thing that is important is that the effort that one has to put in eavesdropping has to dwarf the profits, as this would take the drive away for most attackers. The ease of the attacks will also be a factor for the amount of abuse, and since the attacks that are currently known against quantum key distribution systems are very advanced attacks and have been made increasingly hard by the producers, the possibility that they will be abused is small enough to deem the current quantum key distribution solutions secure enough to use.
Chapter 10

Looking forward

The QKD systems which we have seen so far are based on communication over a quantum channel. But, at this point the main weakness is the distance over which the signals can be sent. The greatest distance that has been realized is 100 kilometers. Furthermore, there still is a gap between the theoretical part of the protocols and the practical part of the devices. This means that the devices do not guarantee perfect secrecy. When the devices improve and the key distribution will be safer, then the practical use of QKD systems will probably increase. In some cases the distance of 100 kilometers will be enough, for example between banks or between government buildings. However, for further popularity of QKD in the future, it is necessary that this distance will be increased. On top of that, we want to exchange information with several people simultaneously.

Single-atom-cavity system

Therefore we need a quantum network, i.e. a fully quantum-mechanical analogous to an ordinary telecommunication network. This is a distributed quantum many-body system in which the different stations are called the “nodes” of the network. The first steps in this direction have been taken by Stephan Ritter and his colleagues of the Max Planck Institute of Quantum Optics in Garching, Germany. The quantum network they presented is based on single atoms embedded in optical cavities. These cavities form universal nodes within the network. The atom-cavity systems are capable of receiving, storing, sending and releasing photonic quantum information. In this new set-up the quantum states do not have to be read out and regenerated at each node. Instead, by applying a laser to one atom, it emits a photon which travels to a second node. The atom in this node absorbs the photon and it appeared that the quantum information of the first atom was now transferred to the second atom. To make sure that the atom absorbs the photon, the atom is placed between two highly reflective mirrors 0.5 millimeter apart (the optical cavity). The photons will be reflected by the mirrors through which the chance that the atom absorbs the photon will be very high. This cavity-based approach to quantum networking offers a clear perspective for scalability, so this is a step towards large scale quantum networks.[24]
Free-space quantum cryptography

At the same time also “Free-space quantum cryptography” has developed. Instead of sending the photons through fibres, they are sent through the sky, which is the “free space”. One of the groups that has done research on this subject consists of C. Kurtsiefer and his colleagues from the Ludwig-Maximilian University of Munich. They managed to exchange a key over a free space distance of 23.4 kilometers between two mountains. The light beam of about 50 millimeter, sent by a telescope, is accurately aligned on another telescope. This receiver has a diameter of 25 cm. Weaknesses of this set-up are air-turbulence effects, optical losses and background counts. Nonetheless, according to their results, they conclude that key exchange to near-Earth orbit should become possible. The main goal of their experiment is a global key distribution network. But instead of single-atom-cavities, this network is based on optical links to low-Earth orbit satellites. This would make key exchange over very large distances, and indeed over the whole world, possible.[32]

Figure 10.1: “Free-space quantum cryptography”: the transmitter telescope on the roof at the University of Munich.

Measurement-based quantum cryptography

Another alternative for QKD, which is interesting to mention, is measurement based quantum cryptography. This protocol is totally different from the former protocols described in this report. In this protocol a third person is involved, who performs the measurements. This third person can be anyone, even Eve. This protocol has not been implemented yet, but most of the attacks that are known, probably do not work on this protocol.[31]

Realizing a large-scale quantum network with the technology that is now available remains a challenge. Maybe in the future, we will send all of our messages encrypted with a key agreed on through quantum key distribution.
A. Internal structure of Clavis$^2$
B. Components of the physical implementation

In this appendix all of the components of the physical implementation of the id3100/id3110 Clavis system are explained.\[12\]

- 200 GHz-Bandpass Filter: A filter that passes light of frequencies within a certain range and attenuates light outside that range. This filter will transmit light frequencies of 200 GHz and a certain amount above and below this number.

- Beam splitter (BS): A beam splitter is a device which splits a light beam exactly in two. This property can be implemented in three different ways. One is based on a dichroic mirrored prism. This mirror lets exactly half of the photons through and reflects the other half. The beam of light is then split in two beams of exactly the same intensity.

- Circulator: A circulator is a three- or four-port device. Light that enters any port is rotated to the next port and then transmitted. There’s only one direction of rotation, which is clockwise.

- Coupler 10/90 - Coupler 50/50: A coupler is a device which splits the light beam in two. It is polarization non-sensitive and also bidirectional. The 10/90 ratio means that the beam is split in one beam with 90 percentage of the original intensity and the other beam with the remaining 10 percentage. Accordingly, the 50/50 ratio means that the coupler splits the light beam in two beams of the same intensity.

- Delay line: The delay line is a fibre optic cable which actually delays the photons to correct for backscattering. It provides a clear separation between the scattered light and the photons that travelled the optical path completely, by delaying the last group.

- Detector: A detector can “detect” a photon, which means that when a photon comes into the device it gives a “click”. A more detailed explanation can be found in chapter 7, section 7.3.

- Faraday mirror: A Faraday mirror consists of a retro mirror and a Faraday rotator situated in front of the mirror. Each time the light passes through the rotator, it’s polarization gets rotated by 45 degrees. Because of the non-reciprocal character of
the rotator, the state of polarization is rotated by 90 degrees with respect to the original signal, when the light comes out.

- **Laser**: A laser is a device that emits monochromatic electromagnetic radiation. A laser is able to produce a thin coherent light beam. This light beam also barely diverges or converges. Due to these properties, a laser is very suitable for a QKD device. A more detailed explanation can be found in chapter 7, section 7.1.

- **Phase modulator**: A phase modulator can change the phase angle of a wave. The device can be installed in a way that the phase of every photon, despite of the current phase of each photon, will be rotated by a specific desired phase.

- **Polarizing beam splitter (PBS)**: A polarizing beam splitter resembles a normal beam splitter, except that the two out-coming beams differ in linear polarization. In the ideal case both out-coming beams would be fully polarized, with orthogonal polarizations. This means that the PBS alters the polarization of a photon if this polarization is not equal to one of the out-coming linear polarizations.

- **Variable attenuator**: An optical attenuator is a device that reduces the power level of an optical signal, in this case in an optical fibre. This means that the amount of photons in the light beam is reduced. Variable attenuators generally use a variable neutral density filter to do this.
Bibliography


