



Number Theory B

1. If you multiply all positive integer factors of 24, you get 24^x . Find x .
2. How many positive integers n are there such that $n + 2$ divides $(n + 18)^2$?
3. Find the number of pairs of integers x, y with different parities such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{2520}$.
4. Find the sum of the reciprocals of the positive integral factors of 84.
5. For how many integers $x \in [0, 2007]$ is $\frac{6x^3+53x^2+61x+7}{2x^2+17x+15}$ reducible?
6. Find the last three digits of
$$2008^{2007^{\dots^{2^1}}}$$
.
7. Find the largest integer n which equals the product of its leading digit and the sum of its digits.
8. In how many ways can $1 + 2 + \dots + 2007$ be expressed as a sum of consecutive positive integers?
9. A positive integer is called *squarefree* if its only perfect square factor is 1. Call a set of positive integers *squarefreeful* if each product of two of its elements is squarefree, and *squarefreefullest* if no positive integer less than the maximum element of the set can be added while preserving the set's squarefreefulness. What is the minimum number of elements in a squarefreefullest set containing 31?
10. Let F_n be the Fibonacci numbers, defined by $F_0 = 0$, $F_1 = 1$, and $F_n = F_{n-1} + F_{n-2}$. For each i , $1 \leq i \leq 200$, we calculate the greatest common divisor g_i of f_i and f_{2007} . What is the sum of the distinct values of g_i ?