# Team

1. If

$$\begin{aligned} x+y+\quad z+w \quad &= 20\\ y+\quad 2z-3w \quad &= 28\\ x-2y+\quad z \quad &= 36\\ -7x-y+\quad 5z+3w \quad &= 84 \end{aligned}$$

then what is $(x, y, z, w)$?

2. Find the biggest non-integer $x$ such that $(x+2)^2+(x+3)^3+(x+4)^4=2$.

3. In triangle $ABC$, let $O$ and $I_A$ be the centers of the circumcircle and the circle tangent to $AB$ and $AC$ and externally tangent to $BC$, and let $R$ and $R_A$ be their radii. Find $\frac{I_AA \cdot I_AB \cdot I_AC}{R \cdot R_A^2}$.

4. Find all $x$ such that $6^x + 27^{x-1} = 8^x - 1$.

5. Integers $x_1, x_2, \ldots, x_{100}$ satisfy

$$\frac{1}{\sqrt{x_1}}+\frac{1}{\sqrt{x_2}}+\cdots+\frac{1}{\sqrt{x_{100}}}=20.$$

Find $\prod_{i \neq j}(x_i - x_j)$.

6. If integers $a$, $b$, $c$, and $d$ satisfy $bc+ad=ac+2bd=1$, find all possible values of $\frac{a^2+c^2}{b^2+d^2}$.

7. In triangle $ABC$, $Q$ and $R$ are points on segments $AC$ and $AB$, respectively, and $P$ is the intersection of $CR$ and $BQ$. If $AR=RB=CP$ and $CQ=PQ$, find $\angle BRC$.

8. Find the biggest $n<2007$ such that there exists a partition of the integers from 1 to $n$ into two sets the sums of the squares of whose elements are equal.

9. Find $p+r$ if $p$ and $q$ are primes and $r$ is a positive integer such that

$$(r^2+pr+1)(r^2+(p^2-q)r-p)=pq.$$

10. If $x$, $y$, and $z$ are real numbers such that $x^2+z^2=1$ and $y^2+2y(x+z)=6$, find the maximum value of $y(z-x)$.