Algebra A

1. If a, b, and c are real numbers such that $a+b+c=6$ and $ab+bc+ca=9$, find the sum of all possible values of the expression $\lfloor a \rfloor + \lfloor b \rfloor + \lfloor c \rfloor$.

2. Positive reals p and q are such that the graph of $y=x^2-2px+q$ does not intersect the x-axis. Find q if there is a unique pair of points A, B on the graph with AB parallel to the x-axis and $\angle AOB = \frac{\pi}{2}$, where O is the origin.

3. For how many rational numbers p is the area of the triangle formed by the intercepts and vertex of $f(x)=-x^2+4px-p+1$ an integer?

4. Find the all values of a such that $x^6-6x^5+12x^4+ax^3+12x^2-6x+1$ is nonnegative for all real x.

5. Find the values of a such that $\log(ax+1)=\log(x-a)+\log(2-x)$ has a unique real solution.

6. If a, b, c, and d are reals with $a \geq b \geq c \geq d \geq 0$ and $b(b-a)+c(c-b)+d(d-c) \leq 2-\frac{a^2}{2}$, find the minimum value of the expression

$$\frac{1}{b+2006c-2006d}+\frac{1}{a+2006b-2006c-d}+\frac{1}{2007a-2006b-c+d}+\frac{1}{a-b+c+2006d}.$$

7. Given two sequences x_n and y_n defined by $x_0=y_0=7$,

$$x_n = 4x_{n-1}+3y_{n-1}, \text{ and}$$

$$y_n = 3y_{n-1}+2x_{n-1},$$

Find $\lim_{n\to\infty} \frac{x_n}{y_n}$.

8. $f(x)=x^3+3x^2-1$. Find the number of real solutions of $f(f(x))=0$.

9. Find the value of $x+y$ for which the expression

$$\frac{6x^2}{y^6}+\frac{6y^2}{x^6}+9x^2y^2+\frac{4}{x^6y^6}$$

is minimized.

10. Find the real root of x^5+5x^3+5x-1. Hint: Let $x=u+k/u$.