1. If $a$, $b$, and $c$ are real numbers such that $a + b + c = 6$ and $ab + bc + ca = 9$, find the sum of all possible values of the expression $|a| + |b| + |c|$.

2. Positive reals $p$ and $q$ are such that the graph of $y = x^2 - 2px + q$ does not intersect the $x$-axis. Find $q$ if there is a unique pair of points $A$, $B$ on the graph with $AB$ parallel to the $x$-axis and $\angle AOB = \frac{\pi}{2}$, where $O$ is the origin.

3. For how many rational numbers $p$ is the area of the triangle formed by the intercepts and vertex of $f(x) = -x^2 + 4px - p + 1$ an integer?

4. Find the all values of $a$ such that $x^6 - 6x^5 + 12x^4 + ax^3 + 12x^2 - 6x + 1$ is nonnegative for all real $x$.

5. Find the values of $a$ such that $\log(ax + 1) = \log(x - a) + \log(2 - x)$ has a unique real solution.

6. If $a$, $b$, $c$, and $d$ are reals with $a \geq b \geq c \geq d \geq 0$ and $b(b - a) + c(c - b) + d(d - c) \leq 2 - \frac{x^2}{2}$, find the minimum value of the expression

$$\frac{1}{b + 2006c - 2006d} + \frac{1}{a + 2006b - 2006c - d} + \frac{1}{2007a - 2006b - c + d} + \frac{1}{a - b + c + 2006d}.$$

7. Given two sequences $x_n$ and $y_n$ defined by $x_0 = y_0 = 7$, $x_n = 4x_{n-1} + 3y_{n-1}$, and $y_n = 3y_{n-1} + 2x_{n-1}$, find $\lim_{n \to \infty} \frac{x_n}{y_n}$.

8. $f(x) = x^3 + 3x^2 - 1$. Find the number of real solutions of $f(f(x)) = 0$.

9. Find the value of $x + y$ for which the expression

$$\frac{6x^2}{y^6} + \frac{6y^2}{x^6} + 9x^2y^2 + \frac{4}{x^6y^6}$$

is minimized.

10. Find the real root of $x^5 + 5x^3 + 5x - 1$. Hint: Let $x = u + k/u$. 