1. Triangle $ABC$ has $AC = 3$, $BC = 5$, $AB = 7$. A circle is drawn internally tangent to the circumcircle of $ABC$ at $C$, and tangent to $AB$. Let $D$ be its point of tangency with $AB$. Find $BD - DA$.

![Diagram]

2. A, B, C, and D are all on a circle, and ABCD is a convex quadrilateral. If $AB = 13$, $BC = 13$, $CD = 37$, and $AD = 47$, what is the area of ABCD?

3. Points $P_1$, $P_2$, $P_3$, and $P_4$ are $(0,0)$, $(10,20)$, $(5,15)$, and $(12,-6)$, respectively. For what point $P \in \mathbb{R}^2$ is the sum of the distances from $P$ to the other 4 points minimal?

4. Find $\frac{\text{area}(CDF)}{\text{area}(CEF)}$ in the figure.

![Diagram]

5. $A$ and $B$ are on a circle of radius 20 centered at $C$, and $\angle ACB = 60^\circ$. $D$ is chosen so that $D$ is also on the circle, $\angle ACD = 160^\circ$, and $\angle DCB = 100^\circ$. Let $E$ be the intersection of lines $AC$ and $BD$. What is $DE$?

6. A sphere of radius $\sqrt{85}$ is centered at the origin in three dimensions. A tetrahedron with vertices at integer lattice points is inscribed inside the sphere. What is the maximum possible volume of this tetrahedron?
7. A set of points $P_i$ covers a polygon if for every point in the polygon, a line can be drawn inside the polygon to at least one $P_i$. Points $A_1, A_2, \ldots, A_n$ in the plane form a 2007-gon, not necessarily convex. Find the minimum value of $n$ such that for any such polygon, we can pick $n$ points inside it that cover the polygon.

8. What is the area of the region defined by $x^2 + 3y^2 \leq 4$ and $y^2 + 3x^2 \leq 4$?

9. There are four spheres each of radius 1 whose centers form a triangular pyramid where each side has length 2. There is a 5th sphere which touches all four other spheres and has radius less than 1. What is its radius?

10. In triangle $ABC$ with $AB \neq AC$, points $N \in CA$, $M \in AB$, $P \in BC$, and $Q \in BC$ are chosen such that $MP \parallel AC$, $NQ \parallel AB$, $\frac{BP}{AB} = \frac{CQ}{AC}$, and $A, M, Q, P, N$ are concyclic. Find $\angle BAC$.