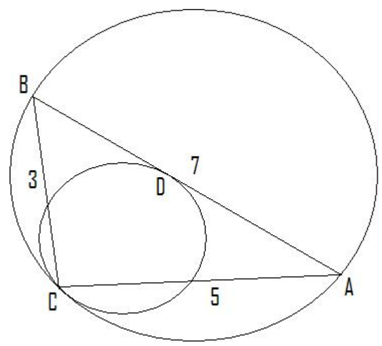
Geometry A

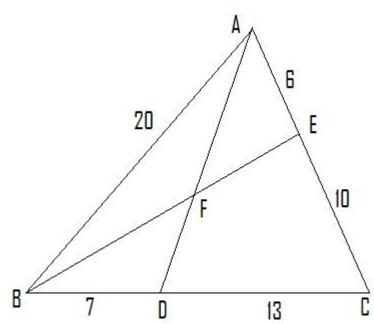
1. Triangle ABC has $AC = 3$, $BC = 5$, $AB = 7$. A circle is drawn internally tangent to the circumcircle of ABC at C, and tangent to AB. Let D be its point of tangency with AB. Find $BD - DA$.



2. A,B,C, and D are all on a circle, and ABCD is a convex quadrilateral. If AB = 13, BC = 13, CD = 37, and AD = 47, what is the area of ABCD?

3. Points P_1, P_2, P_3, and P_4 are $(0, 0)$, $(10, 20)$, $(5, 15)$, and $(12, -6)$, respectively. For what point $P \in \mathbb{R}^2$ is the sum of the distances from P to the other 4 points minimal?

4. Find $\frac{area(CDF)}{area(CEF)}$ in the figure.



5. A and B are on a circle of radius 20 centered at C, and $\angle ACB = 60°$. D is chosen so that D is also on the circle, $\angle ACD = 160°$, and $\angle DCB = 100°$. Let E be the intersection of lines AC and BD. What is DE?

6. A sphere of radius $\sqrt{85}$ is centered at the origin in three dimensions. A tetrahedron with vertices at integer lattice points is inscribed inside the sphere. What is the maximum possible volume of this tetrahedron?

Geometry A

7. A set of points P_i *covers* a polygon if for every point in the polygon, a line can be drawn inside the polygon to at least one P_i. Points A_1, A_2, …, A_n in the plane form a 2007-gon, not necessarily convex. Find the minimum value of n such that for any such polygon, we can pick n points inside it that cover the polygon.

8. What is the area of the region defined by $x^2 + 3y^2 \leq 4$ and $y^2 + 3x^2 \leq 4$?

9. There are four spheres each of radius 1 whose centers form a triangular pyramid where each side has length 2. There is a 5th sphere which touches all four other spheres and has radius less than 1. What is its radius?

10. In triangle ABC with $AB \neq AC$, points $N \in CA$, $M \in AB$, $P \in BC$, and $Q \in BC$ are chosen such that $MP||AC$, $NQ||AB$, $\frac{BP}{AB} = \frac{CQ}{AC}$, and A, M, Q, P, N are concyclic. Find $\angle BAC$.