1. A pirate ship spots, 10 nautical miles to the east, an oblivious caravel sailing 60° south of west at a steady 12 nm/hour. What is the minimum speed that the pirate ship must maintain at to be able to catch the caravel?

2. A black witch’s hat is in the classic shape of a cone on top of a circular brim. The cone has a slant height of 18 inches and a base radius of 3 inches. The brim has a radius of 5 inches. What is the total surface area of the hat?

3. Suppose that ABCD is a rectangle with sides of length 12 and 18. Let S be the region of points contained in ABCD which are closer to the center of the rectangle than to any of its vertices. Find the area of S.

4. ABCDE is a regular pentagon (with vertices in that order) inscribed in a circle of radius 1. Find $AB \cdot AC$.

5. Four circles of radius 1 are each tangent to two sides (line segments) of a square and externally tangent to a circle of radius 3. What is the area of the space that is inside the square but not contained in any of the circles?

6. Triangle $ABC$ has $AC = 3$, $BC = 5$, $AB = 7$. A circle is drawn internally tangent to the circum-circle of $ABC$ at $C$, and tangent to $AB$. Let $D$ be its point of tangency with $AB$. Find $BD - DA$.

7. A,B,C, and D are all on a circle, and ABCD is a convex quadrilateral. If $AB = 13$, $BC = 13$, $CD = 37$, and $AD = 47$, what is the area of ABCD?

8. Points $P_1$, $P_2$, $P_3$, and $P_4$ are $(0,0)$, $(10,20)$, $(5,15)$, and $(12,−6)$, respectively. For what point $P \in \mathbb{R}^2$ is the sum of the distances from $P$ to the other 4 points minimal?
9. Find \( \frac{\text{area}(CDF)}{\text{area}(CEF)} \) in the figure.

10. \( A \) and \( B \) are on a circle of radius 20 centered at \( C \), and \( \angle ACB = 60^\circ \). \( D \) is chosen so that \( D \) is also on the circle, \( \angle ACD = 160^\circ \), and \( \angle DCB = 100^\circ \). Let \( E \) be the intersection of lines \( AC \) and \( BD \). What is \( DE \)?