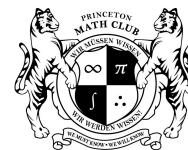


# PUMaC 2008-9



## Number Theory B

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- (2 points) What is the remainder, in base 10, when  $24_7 + 364_7 + 43_7 + 12_7 + 3_7 + 1_7$  is divided by 6?
- (2 points) How many zeros are there at the end of  $792!$  when written in base 10?
- (3 points) Find all integral solutions to  $x^y - y^x = 1$ .
- (3 points) Find the largest integer  $n$ , where  $2009^n$  divides  $2008^{2009^{2010}} + 2010^{2009^{2008}}$ .
- (4 points) How many integers  $n$  are there such that  $0 \leq n \leq 720$  and  $n^2 \equiv 1 \pmod{720}$ ?
- (4 points)  $f(n)$  is the sum of all integers less than  $n$  and relatively prime to  $n$ . Find all integers  $n$  such that there exist integers  $k$  and  $l$  such that  $f(n^k) = n^l$ .
- (5 points) In this problem, we consider only polynomials with integer coefficients. Call two polynomials  $p$  and  $q$  *really close* if  $p(2k+1) \equiv q(2k+1) \pmod{2^{10}}$  for all  $k \in \mathbb{Z}^+$ . Call a polynomial  $p$  *partial credit* if no polynomial of lesser degree is *really close* to it. What is the maximum possible degree of partial credit?
- (5 points) What is the largest integer which cannot be expressed as  $2008x + 2009y + 2010z$  for some positive integers  $x$ ,  $y$ , and  $z$ ?
- (7 points) Find all sets of three primes  $p$ ,  $q$ , and  $r$  such that  $p + q = r$  and  $(r - p)(q - p) - 27p$  is a perfect square.
- (7 points) What is the smallest number  $n$  such that you can choose  $n$  distinct odd integers  $a_1, a_2, \dots, a_n$ , none of them 1, with  $\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n} = 1$ ?