1. Find the root that the following three polynomials have in common:

\[ x^3 + 41x^2 - 49x - 2009 \]
\[ x^3 + 5x^2 - 49x - 245 \]
\[ x^3 + 39x^2 - 117x - 1435 \]

2. Given that \( P(x) \) is the least degree polynomial with rational coefficients such that

\[ P(\sqrt{2} + \sqrt{3}) = \sqrt{2}, \]

find \( P(10) \).

3. Let \( x_1, x_2, \ldots, x_{10} \) be non-negative real numbers such that \( \frac{x_1^2}{1} + \frac{x_2^2}{2} + \ldots + \frac{x_{10}^2}{10} \leq 9 \). Find the maximum possible value of \( \frac{x_1^2}{1} + \frac{x_2^2}{2} + \ldots + \frac{x_{10}^2}{10} \).

4. Find the smallest positive \( \alpha \) (in degrees) for which all the numbers

\[ \cos \alpha, \cos 2\alpha, \ldots, \cos 2^n \alpha, \ldots \]

are negative.

5. Find the maximal positive integer \( n \), so that for any real number \( x \) we have \( \sin^n x + \cos^n x \geq \frac{1}{n} \).

6. Find the number of functions \( f : \mathbb{Z} \rightarrow \mathbb{Z} \) for which \( f(h + k) + f(hk) = f(h)f(k) + 1 \), for all integers \( h \) and \( k \).

7. Let \( x_1, x_2, \ldots, x_n \) be a sequence of integers, such that \(-1 \leq x_i \leq 2\), for \( i = 1, 2, \ldots, n \), \( x_1 + x_2 + \cdots + x_n = 7 \) and \( x_1^8 + x_2^8 + \ldots + x_n^8 = 2009 \). Let \( m \) and \( M \) be the minimal and maximal possible value of \( x_1^8 + x_2^8 + \ldots + x_n^8 \), respectively. Find the \( \frac{M}{m} \). Round your answer to nearest integer, if necessary.

8. The real numbers \( x, y, z, \) and \( t \) satisfy the following equation:

\[ 2x^2 + 4xy + 3y^2 - 2xz - 2yz + z^2 + 1 = t + \sqrt{y + z - t} \]

Find 100 times the maximum possible value for \( t \).