Combinatorics A

1. Find the number of subets of $\{1,2,\ldots,7\}$ that do not contain two consecutive integers.

2. It is known that a certain mechanical balance can measure any object of integer mass anywhere between 1 and 2009 (both included). This balance has k weights of integral values. What is the minimum k for which there exist weights that satisfy this condition?

3. How many strings of ones and zeroes of length 10 are there such that there is an even number of ones, and no zero follows another zero?

4. We divide up the plane into disjoint regions using a circle, a rectangle and a triangle. What is the greatest number of regions that we can get?

5. There are n players in a round-robin ping-pong tournament (i.e. every two persons will play exactly one game). After some matches have been played, it is known that the total number of matches that have been played among any $n-2$ people is equal to 3^k (where k is a fixed integer). Find the sum of all possible values of n.

6. We have a 6×6 square, partitioned into 36 unit squares. We select some of these unit squares and draw some of their diagonals, subject to the condition that no two diagonals we draw have any common points. What is the maximal number of diagonals that we can draw?

7. We randomly choose 5 distinct positive integers less than or equal to 90. What is the floor of 10 times the expected value of the fourth largest number?

8. Taotao wants to buy a bracelet. The bracelets have 7 different beads on them, arranged in a circle. Two bracelets are the same if one can be rotated or flipped to get the other. If she can choose the colors and placement of the beads, and the beads come in orange, white, and black, how many possible bracelets can she buy?