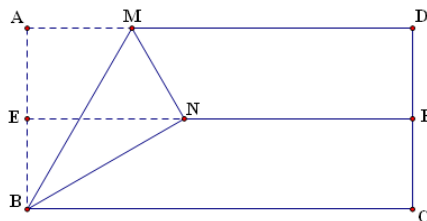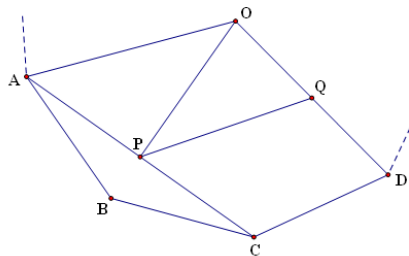# PUMaC 2009

Geometry A

1. A rectangular piece of paper $ABCD$ has sides of lengths $AB = 1$, $BC = 2$. The rectangle is folded in half such that AD coincides with BC and EF is the folding line. Then fold the paper along a line BM such that the corner A falls on line EF. How large, in degrees, is $\angle ABM$?

2. Tetrahedron $ABCD$ havs sides of lengths, in increasing order, 7, 13, 18, 27, 36, 41. If $AB = 41$, then what is the length of CD?

3. A polygon is called concave if it has at least one angle strictly greater than $180°$. What is the maximum number of symmetries that an 11-sided concave polygon can have?

4. In the following diagram (not to scale), A, B, C, D are four consecutive vertices of an 18-sided regular polygon with center O. Let P be the midpoint of AC and Q be the midpoint of DO. Find $\angle OPQ$ in degrees.
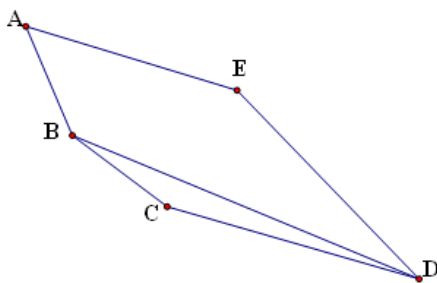

5. Lines l and m are perpendicular. Line l partitions a convex polygon into two parts of equal area, and partitions the projection of the polygon onto m into two line segments of length a and b respectively. Determine the maximum value of $\lfloor \frac{1000a}{b} \rfloor$. (The floor notation $\lfloor x \rfloor$ denotes largest integer not exceeding x)

6. Consider the solid with 4 triangles and 4 regular hexagons as faces, where each triangle borders 3 hexagons, and all the sides are of length 1. Compute the *square* of the volume of the solid. Express your result in reduced fraction and concatenate the numerator with the denominator (e.g., if you think that the square is $\frac{1734}{274}$, then you would submit 1734274).

(Please Turn Over)

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7. You are given a convex pentagon $ABCDE$ with $AB = BC$, $CD = DE$, $\angle ABC = 150°$, $\angle BCD = 165°$, $\angle CDE = 30°$, $BD = 6$. Find the area of this pentagon. Round your answer to the nearest integer if necessary.

8. Consider $\triangle ABC$ and a point M in its interior so that $\angle MAB = 10°$, $\angle MBA = 20°$, $\angle MCA = 30°$ and $\angle MAC = 40°$. What is $\angle MBC$?