1. You are given that
\[ 17! = 355687ab8096000 \]
for some digits \( a \) and \( b \). Find the two-digit number \( \overline{ab} \) that is missing above.

2. Find the number of ordered pairs \( (a, b) \) of positive integers that are solutions of the following equation:
\[ a^2 + b^2 = ab(a + b) \]

3. Find the sum of all prime numbers \( p \) which satisfy
\[ p = a^4 + b^4 + c^4 - 3 \]
for some primes (not necessarily distinct) \( a, b \) and \( c \).

4. Find the sum of all integers \( x \) for which there is an integer \( y \), such that \( x^3 - y^3 = xy + 61 \).

5. Suppose that for some positive integer \( n \), the first two digits of \( 5^n \) and \( 2^n \) are identical. Suppose the first two digits are \( a \) and \( b \) in this order. Find the two-digit number \( \overline{ab} \).

6. Let \( s(m) \) denote the sum of the digits of the positive integer \( m \). Find the largest positive integer that has no digits equal to zero and satisfies the equation
\[ 2^{s(n^2)} = s(n^2) \]

7. Let \( S = \{ p/q \mid q \leq 2009, p/q < 1257/2009, p, q \in \mathbb{N} \} \). If the maximum element of \( S \) is \( p_0/q_0 \) in reduced form, find \( p_0 + q_0 \).

8. Find the largest positive integer \( k \) such that \( \phi(\sigma(2^k)) = 2^k \). (\( \phi(n) \) denotes the number of positive integers that are smaller than \( n \) and relatively prime to \( n \), and \( \sigma(n) \) denotes the sum of divisors of \( n \)). As a hint, you are given that \( 641|2^{32} + 1 \).