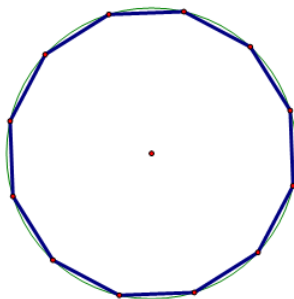# PUMaC 2009

Geometry B

1. Find 100 times the area of a regular dodecagon inscribed in a unit circle. Round your answer to the nearest integer if necessary.

2. A triangle has sides of lengths 5, 6, 7. What is 60 times the square of the radius of the inscribed circle?

3. A rectangular piece of paper $ABCD$ has sides of lengths $AB = 1$, $BC = 2$. The rectangle is folded in half such that AD coincides with BC and EF is the folding line. Then fold the paper along a line BM such that the corner A falls on line EF. How large, in degrees, is $\angle ABM$?
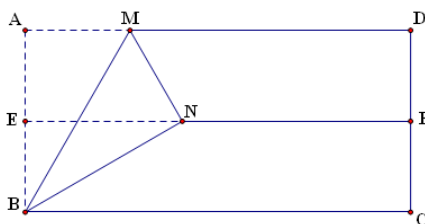

4. Tetrahedron $ABCD$ has sides of lengths, in increasing order, 7, 13, 18, 27, 36, 41. If $AB = 41$, then what is the length of CD?

5. A polygon is called concave if it has at least one angle strictly greater than $180°$. What is the maximum number of symmetries that an 11-sided concave polygon can have? (A *symmetry* of a polygon is a way to rotate or reflect the plane that leaves the polygon unchanged.)

(Please Turn Over)

6. In the following diagram (not to scale), A, B, C, D are four consecutive vertices of an 18-sided regular polygon with center O. Let P be the midpoint of AC and Q be the midpoint of DO. Find $\angle OPQ$ in degrees.
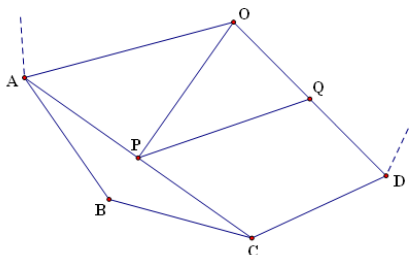
7. Lines l and m are perpendicular. Line l partitions a convex polygon into two parts of equal area, and partitions the projection of the polygon onto m into two line segments of length a and b respectively. Determine the maximum value of $\lfloor \frac{1000a}{b} \rfloor$. (The floor notation $\lfloor x \rfloor$ denotes largest integer not exceeding x)

8. Consider the solid with 4 triangles and 4 regular hexagons as faces, where each triangle borders 3 hexagons, and all the sides are of length 1. Compute the *square* of the volume of the solid. Express your result in reduced fraction and concatenate the numerator with the denominator (e.g., if you think that the square is $\frac{1734}{274}$, then you would submit 1734274).