Number Theory B

1. Find the number of pairs of integers $x$ and $y$ such that $x^2 + xy + y^2 = 28$.

2. Suppose you are given that for some positive integer $n$, $1! + 2! + \ldots + n!$ is a perfect square. Find the sum of all possible values of $n$.

3. You are given that

$$17! = 355687ab8096000$$

for some digits $a$ and $b$. Find the two-digit number $\overline{ab}$ that is missing above.

4. Find the number of ordered pairs $(a, b)$ of positive integers that are solutions of the following equation:

$$a^2 + b^2 = ab(a + b)$$

5. Find the sum of all prime numbers $p$ which satisfy

$$p = a^4 + b^4 + c^4 - 3$$

for some primes (not necessarily distinct) $a$, $b$, and $c$.

6. Find the sum of all integers $x$ for which there is an integer $y$, such that $x^3 - y^3 = xy + 61$.

7. Suppose that for some positive integer $n$, the first two digits of $5^n$ and $2^n$ are identical. Suppose the first two digits are $a$ and $b$ in this order. Find the two-digit number $\overline{ab}$.

8. Let $s(m)$ denote the sum of the digits of the positive integer $m$. Find the largest positive integer that has no digits equal to zero and satisfies the equation

$$2^{s(n)} = s(n^2)$$