1. Find the sum of the coefficients of the polynomial \((63x - 61)^4\).

2. Calculate \(\sum_{n=1}^{\infty} \left(\lfloor \sqrt[4]{2010} \rfloor - 1\right)\) where \(\lfloor x \rfloor\) is the largest integer less than or equal to \(x\).

3. Let \(S\) be the sum of all real \(x\) such that \(4^x = x^4\). Find the nearest integer to \(S\).

4. Define \(f(x) = x + \sqrt{x + \sqrt{x + \sqrt{x + \ldots}}}\). Find the smallest integer \(x\) such that \(f(x) \geq 50\sqrt{x}\).

5. Let \(f(x) = 3x^3 - 5x^2 + 2x - 6\). If the roots of \(f\) are given by \(\alpha, \beta, \) and \(\gamma\), find 
\[
\left(\frac{1}{\alpha - 2}\right)^2 + \left(\frac{1}{\beta - 2}\right)^2 + \left(\frac{1}{\gamma - 2}\right)^2.
\]

6. Assume that \(f(a + b) = f(a) + f(b) + ab\), and that \(f(75) - f(51) = 1230\). Find \(f(100)\).

7. The expression \(\sin 2^\circ \sin 4^\circ \sin 6^\circ \cdots \sin 90^\circ\) is equal to \(p\sqrt{5}/2^{30}\), where \(p\) is an integer. Find \(p\).

8. Let \(p\) be a polynomial with integer coefficients such that \(p(15) = 6\), \(p(22) = 1196\), and \(p(35) = 26\). Find an integer \(n\) such that \(p(n) = n + 82\).