PUMaC 2010

Number Theory A

1. Find the smallest positive integer n such that $n^4 + (n+1)^4$ is composite.
2. Find the largest positive integer n such that $\sigma(n) = 28$, where $\sigma(n)$ is the sum of the divisors of n, including n.
3. Find the sum of the first 5 positive integers n such that $n^2 - 1$ is the product of 3 distinct primes.
4. Find the largest positive integer n such that $n\varphi(n)$ is a perfect square. ($\varphi(n)$ is the number of integers k, $1 \le k \le n$ that are relatively prime to n)
5. Given that x, y are positive integers with $x(x+1)|y(y+1)$, but neither x nor $x+1$ divides either of y or $y+1$, and $x^2 + y^2$ as small as possible, find $x^2 + y^2$.
6. Find the numerator of
$$\frac{1010\overbrace{11\ldots11}^{2011\text{ ones}}0101}{1100\underbrace{11\ldots11}_{2011\text{ ones}}0011}$$
when reduced.
7. Let n be the number of polynomial functions from the integers modulo 2010 to the integers modulo 2010. n can be written as $n = p_1 p_2 \cdots p_k$, where the p_is are (not necessarily distinct) primes. Find $p_1 + p_2 + \cdots + p_n$.
8. A consecutive pythagorean triple is a pythagorean triple of the form $a^2 + (a+1)^2 = b^2$, a and b positive integers. Given that a, $a+1$, and b form the third consecutive pythagorean triple, find a.