1. Let the operation $\star$ be defined by $x \star y = y^x - xy$. Calculate $(3 \star 4) - (4 \star 3)$.

2. Let $p(x) = x^2 + x + 1$. Find the fourth smallest prime $q$ such that $p(n)$ is divisible by $q$ for some integer $n$.

3. Write $\frac{1}{\sqrt{2} - 1} = a + b\sqrt{2} + c\sqrt{4} + d\sqrt{8} + e\sqrt{16}$, with $a, b, c, d, e$ integers. Find $a^2 + b^2 + c^2 + d^2 + e^2$.

4. Let $S$ be the sum of all real $x$ such that $4^x = x^4$. Find the nearest integer to $S$.

5. Let $x$ be a real root of the polynomial $p(x) = x^3 - 3x + 3$. Find $x^9 + 81x^2$.

6. Define $f(x) = x + \sqrt{x + \sqrt{x + \sqrt{x + \cdots}}}$. Find the smallest integer $x$ such that $f(x) \geq 50\sqrt{x}$.

7. Let $f$ be a function such that $f(x) + f(x + 1) = 2^x$ and $f(0) = 2010$. Find the last two digits of $f(2010)$.

8. The expression $\sin 2^\circ \sin 4^\circ \sin 6^\circ \cdots \sin 90^\circ$ is equal to $p\sqrt{5}/2^{50}$, where $p$ is an integer. Find $p$. 
