



## Algebra B

1. Let the operation  $\star$  be defined by  $x \star y = y^x - xy$ . Calculate  $(3 \star 4) - (4 \star 3)$ .
2. Let  $p(x) = x^2 + x + 1$ . Find the fourth smallest prime  $q$  such that  $p(n)$  is divisible by  $q$  for some integer  $n$ .
3. Write  $\frac{1}{\sqrt[5]{2}-1} = a + b\sqrt[5]{2} + c\sqrt[5]{4} + d\sqrt[5]{8} + e\sqrt[5]{16}$ , with  $a, b, c, d,$  and  $e$  integers. Find  $a^2 + b^2 + c^2 + d^2 + e^2$ .
4. Let  $S$  be the sum of all real  $x$  such that  $4^x = x^4$ . Find the nearest integer to  $S$ .
5. Let  $x$  be a real root of the polynomial  $p(x) = x^3 - 3x + 3$ . Find  $x^9 + 81x^2$ .
6. Define  $f(x) = x + \sqrt{x + \sqrt{x + \sqrt{x + \sqrt{x + \dots}}}}$ . Find the smallest integer  $x$  such that  $f(x) \geq 50\sqrt{x}$ .
7. Let  $f$  be a function such that  $f(x) + f(x+1) = 2^x$  and  $f(0) = 2010$ . Find the last two digits of  $f(2010)$ .
8. The expression  $\sin 2^\circ \sin 4^\circ \sin 6^\circ \cdots \sin 90^\circ$  is equal to  $p\sqrt{5}/2^{50}$ , where  $p$  is an integer. Find  $p$ .