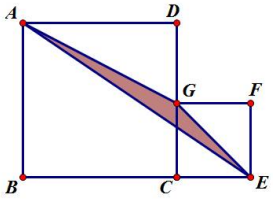
PUMaC 2010

Geometry B

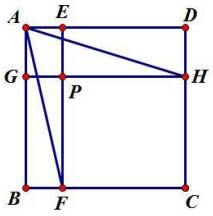
1. In a polygon, every external angle is one sixth of its corresponding internal angle. How many sides does the polygon have?

2. On rectangular coordinates, point $A = (1, 2)$, $B = (3, 4)$. $P = (a, 0)$ is on x-axis. Given that P is chosen such that $AP + PB$ is minimized, compute $60a$.

3. As in the following diagram, square $ABCD$ and square $CEFG$ are placed side by side (i.e. C is between B and E and G is between C and D). Now $CE = 14$, $AB > 14$, compute the minimal area of $\triangle AEG$.

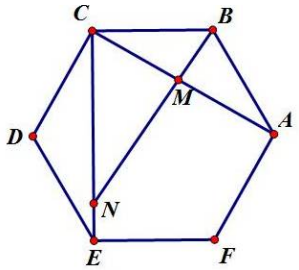


4. Unit square $ABCD$ is divided into four rectangles by EF and GH, with $BF = \frac{1}{4}$. EF is parallel to AB and GH parallel to BC. EF and GH meet at point P. Suppose $BF + DH = FH$, calculate the nearest integer to the degree of $\angle FAH$.

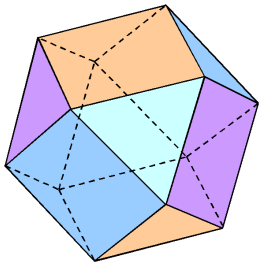


5. In a rectangular plot of land, a man walks in a very peculiar fashion. Labeling the corners $ABCD$, he starts at A and walks to C. Then, he walks to the midpoint of side AD, say A_1. Then, he walks to the midpoint of side CD say C_1, and then the midpoint of A_1D which is A_2. He continues in this fashion, indefinitely. The total length of his path if $AB = 5$ and $BC = 12$ is of the form $a + b\sqrt{c}$. Find $\frac{abc}{4}$.

6. In regular hexagon $ABCDEF$, AC, CE are two diagonals. Points M, N are on AC, CE respectively and satisfy $AC : AM = CE : CN = r$. Suppose B, M, N are collinear, find $100r^2$.



7. A cuboctahedron is a solid with 6 square faces and 8 equilateral triangle faces, with each edge adjacent to both a square and a triangle (see picture). Suppose the ratio of the volume of an octahedron to a cuboctahedron with the same side length is r. Find $100r^2$.



8. Point P is in the interior of $\triangle ABC$. The side lengths of ABC are $AB = 7$, $BC = 8$, $CA = 9$. The three foots of perpendicular lines from P to sides BC, CA, AB are D, E, F respectively. Suppose the minimal value of $\frac{BC}{PD} + \frac{CA}{PE} + \frac{AB}{PF}$ can be written as $\frac{a}{b}\sqrt{c}$, where $\gcd(a, b) = 1$ and c is square free, calculate abc.