



## Number Theory B

1. Find the positive integer less than 18 with the most positive divisors.

2. Let  $f(n)$  be the sum of the digits of  $n$ . Find  $\sum_{n=1}^{99} f(n)$ .

3. Find the smallest positive integer  $n$  such that  $n^4 + (n + 1)^4$  is composite.

4. Find the sum of the first 5 positive integers  $n$  such that  $n^2 - 1$  is the product of 3 distinct primes.

5. Given that  $x$ ,  $y$ , and  $z$  are positive integers such that  $\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = 2$ . Find the sum of all possible  $x$  values.

6. Given that  $x$ ,  $y$  are positive integers with  $x(x + 1) | y(y + 1)$ , but neither  $x$  nor  $x + 1$  divides either of  $y$  or  $y + 1$ , and  $x^2 + y^2$  as small as possible, find  $x^2 + y^2$ .

7. Find the numerator of

$$\frac{1010 \overbrace{11 \dots 11}^{2011 \text{ ones}} 0101}{1100 \underbrace{11 \dots 11}_{2011 \text{ ones}} 0011}$$

when reduced.

8. Let  $N$  be the number of (positive) divisors of  $2010^{2010}$  ending in the digit 2. What is the remainder when  $N$  is divided by 2010?