



## Geometry A

1. [3] Two logs of length 10 are laying on the ground touching each other. Their radii are 3 and 1, and the smaller log is fastened to the ground. The bigger log rolls over the smaller log without slipping, and stops as soon as it touches the ground again. The volume of the set of points swept out by the larger log as it rolls over the smaller one can be expressed as  $n\pi$ , where  $n$  is an integer. Find  $n$ .
2. [3] A rectangular piece of paper has corners labeled  $A, B, C$ , and  $D$ , with  $BC = 80$  and  $CD = 120$ . Let  $M$  be the midpoint of side  $AB$ . The corner labeled  $A$  is folded along line  $MD$  and the corner labeled  $B$  is folded along line  $MC$  until the segments  $AM$  and  $MB$  coincide. Let  $S$  denote the point in space where  $A$  and  $B$  meet. If  $H$  is the foot of the perpendicular from  $S$  to the original plane of the paper, find  $HM$ .
3. [4] Let  $PQ$  and  $PR$  be tangents to a circle  $\omega$  with diameter  $AB$  so that  $A, Q, R, B$  lie on  $\omega$  in that order. Let  $H$  be the projection of  $P$  onto  $AB$  and let  $AR$  and  $PH$  intersect at  $S$ . If  $\angle QPH = 30^\circ$  and  $\angle HPR = 20^\circ$ , find  $\angle ASQ$  in degrees.
4. [4] Let  $ABC$  be a triangle with  $AB = 15, BC = 17, CA = 21$ , and incenter  $I$ . If the circumcircle of triangle  $IBC$  intersects side  $AC$  again at  $P$ , find  $CP$ .
5. [5] Let  $\ell_1$  and  $\ell_2$  be two parallel lines, a distance of 15 apart. Points  $A$  and  $B$  lie on  $\ell_1$  while points  $C$  and  $D$  lie on  $\ell_2$  such that  $\angle BAC = 30^\circ$  and  $\angle ABD = 60^\circ$ . The minimum value of  $AD + BC$  is  $a\sqrt{b}$ , where  $a$  and  $b$  are integers and  $b$  is squarefree. Find  $a + b$ .
6. [6] Let  $\omega_1$  be a circle of radius 6, and let  $\omega_2$  be a circle of radius 5 that passes through the center  $O$  of  $\omega_1$ . Let  $A$  and  $B$  be the points of intersection of the two circles, and let  $P$  be a point on major arc  $AB$  of  $\omega_2$ . Let  $M$  and  $N$  be the second intersections of  $PA$  and  $PB$  with  $\omega_1$ , respectively. Let  $S$  be the midpoint of  $MN$ . As  $P$  ranges over major arc  $AB$  of  $\omega_2$ , the minimum length of segment  $SA$  is  $a/b$ , where  $a$  and  $b$  are positive integers and  $\gcd(a, b) = 1$ . Find  $a + b$ .
7. [7] Let  $ABC$  be a triangle with  $AB = 2, BC = 5, AC = 4$ . Let  $M$  be the projection of  $C$  onto the external angle bisector at vertex  $B$ . Similarly, let  $N$  be the projection of  $B$  onto the external angle bisector at vertex  $C$ . If the ratio of the area of quadrilateral  $BCNM$  to the area of triangle  $ABC$  is  $a/b$ , where  $a$  and  $b$  are positive integers and  $\gcd(a, b) = 1$ , find  $a + b$ .
8. [8] Let  $ABC$  be a triangle with  $\angle BAC = 60^\circ, BA = 2$ , and  $CA = 3$ . A point  $M$  is located inside  $ABC$  such that  $MB = 1$  and  $MC = 2$ . A semicircle tangent to  $MB$  and  $MC$  has its center  $O$  on  $BC$ . Let  $P$  be the intersection of the angle bisector of  $\angle BAC$  and the perpendicular bisector of  $AC$ . If the ratio  $OP/MO$  is  $a/b$ , where  $a$  and  $b$  are positive integers and  $\gcd(a, b) = 1$ , find  $a + b$ .