



Number Theory A

- [3] The only prime factors of an integer n are 2 and 3. If the sum of the divisors of n (including itself) is 1815, find n .
- [3] What is the largest positive integer $n < 1000$ for which there is a positive integer m satisfying

$$\text{lcm}(m, n) = 3m \times \text{gcd}(m, n)?$$

- [4] What is the sum of all primes p such that $7^p - 6^p + 2$ is divisible by 43?
- [4] For how many ordered triplets of three positive integers is it true that their product is four more than twice their sum?
- [5] Let $d(n)$ denote the number of divisors of n (including itself). You are given that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Find $p(6)$, where $p(x)$ is the unique polynomial with rational coefficients satisfying

$$p(\pi) = \sum_{n=1}^{\infty} \frac{d(n)}{n^2}.$$

- [6] Let a and b be positive integers such that $a + bz = x^3 + y^4$ has no solutions for any integers x, y, z , with b as small as possible, and a as small as possible for the minimum b . Find ab .
- [7] Let $\{g_i\}_{i=0}^{\infty}$ be a sequence of positive integers such that $g_0 = g_1 = 1$ and the following recursions hold for every positive integer n :

$$\begin{aligned} g_{2n+1} &= g_{2n-1}^2 + g_{2n-2}^2 \\ g_{2n} &= 2g_{2n-1}g_{2n-2} - g_{2n-2}^2 \end{aligned}$$

Compute the remainder when g_{2011} is divided by 216.

- [8] Calculate the sum of the coordinates of all pairs of positive integers (n, k) such that $k \equiv 0, 3 \pmod{4}$, $n > k$, and $\sum_{i=k+1}^n i^3 = (96^2 \cdot 3 - 1) \left(\sum_{i=1}^k i \right)^2 + 48^2$.