1. [3] What is the largest prime factor of 7999488?

2. [3] Two robots are programmed to communicate numbers using different bases. The first robot states: “I communicate in base 10, which interestingly is a perfect square. You communicate in base 16, which is not a perfect square.” The second robot states: “I find it more interesting that the sum of our bases is the factorial of an integer.” The second robot is referring to the factorial of which integer?

3. [4] The only prime factors of an integer $n$ are 2 and 3. If the sum of the divisors of $n$ (including itself) is 1815, find $n$.

4. [4] What is the largest positive integer $n < 1000$ for which there is a positive integer $m$ satisfying

$$\text{lcm}(m,n) = 3m \times \gcd(m,n)?$$

5. [5] Find the sum of all primes $p$ such that $7^p - 6^p + 2$ is divisible by 43.

6. [6] For how many ordered triplets of three positive integers is it true that their product is four more than twice their sum?

7. [7] Find the sum of all positive integers $k$ with $k \leq 1000$ such that there exists an integer $n > k$ that satisfies

$$\sum_{i=k+1}^{n} i^3 = 3 \left( \sum_{i=1}^{k} i \right)^2.$$

8. [8] Let $d(n)$ denote the number of divisors of $n$ (including itself). You are given that

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}.$$

Find $p(6)$, where $p(x)$ is the unique polynomial with rational coefficients satisfying

$$p(\pi) = \sum_{n=1}^{\infty} \frac{d(n)}{n^2}.$$