PUMaC 2012

Algebra B

1. [**3**] Find the largest n such that the last nonzero digit of $n!$ is 1.

2. [**3**] Define a sequence a_n such that $a_n = a_{n-1} - a_{n-2}$. Let $a_1 = 6$ and $a_2 = 5$. Find $\sum_{n=1}^{1000} a_n$.

3. [**4**] Evaluate $\sqrt[3]{26 + 15\sqrt{3}} + \sqrt[3]{26 - 15\sqrt{3}}$.

4. [**4**] If x, y, and z are real numbers with $\frac{x-y}{z} + \frac{y-z}{x} + \frac{z-x}{y} = 36$, find

$$2012 + \frac{x-y}{z} \cdot \frac{y-z}{x} \cdot \frac{z-x}{y}$$

5. [**5**] Considering all numbers of the form $n = \lfloor \frac{k^3}{2012} \rfloor$, where $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x, and k ranges from 1 to 2012, how many of these n's are distinct?

6. [**6**] Compute

$$\sum_{n=1}^{\infty} \frac{n+1}{n^2(n+2)^2}$$

   Your answer in simplest form can be written as a/b, where a, b are relatively-prime positive integers. Find $a + b$.

7. [**7**] Let f be a polynomial of degree 3 with integer coefficients such that $f(0) = 3$ and $f(1) = 11$. If f has exactly 2 integer roots, how many such polynomials f exist?

8. [**8**] Let a_n be a sequence such that $a_1 = 1$ and $a_{n+1} = \lfloor a_n + \sqrt{a_n} + \frac{1}{2} \rfloor$. What are the last four digits of a_{2012}?