Number Theory A

1. [**3**] If p, q and r are primes with $pqr = 7(p+q+r)$, find $p+q+r$.

2. [**3**] What is the smallest positive integer n such that 2013^n ends in 001 (i.e. the rightmost three digits of 2013^n are 001)?

3. [**4**] Let A be the greatest possible value of a product of positive integers that sums to 2014. Compute the sum of all bases and exponents in the prime factorization of A. For example, if $A = 7 \cdot 11^5$, the answer would be $7 + 11 + 5 = 23$.

4. [**4**] Let d be the greatest common divisor of $2^{30^{10}} - 2$ and $2^{30^{45}} - 2$. Find the remainder when d is divided by 2013.

5. [**5**] Define a "digitized number" as a ten-digit number $a_0 a_1 \ldots a_9$ such that for $k = 0, 1, \ldots, 9$, a_k is equal to the number of times the digit k occurs in the number. Find the sum of all digitized numbers.

6. [**6**] What is the largest positive integer that cannot be expressed as a sum of non-negative integer multiples of $13, 17$ and 23?

7. [**7**] Suppose $P(x)$ is a degree n monic polynomial with integer coefficients such that 2013 divides $P(r)$ for exactly 1000 values of r between 1 and 2013 inclusive. Find the minimum value of n.

8. [**8**] Find the number of primes p between 100 and 200 for which $x^{11} + y^{16} \equiv 2013 \pmod{p}$ has a solution in integers x and y.