PUMaC 2013

Number Theory B

1. [**3**] If p, q and r are primes with $pqr = 7(p+q+r)$, find $p+q+r$.

2. [**3**] What is the smallest positive integer n such that 2013^n ends in 001 (i.e. the rightmost three digits of 2013^n are 001)?

3. [**4**] Find the smallest positive integer x such that
   - x is 1 more than a multiple of 3,
   - x is 3 more than a multiple of 5,
   - x is 5 more than a multiple of 7,
   - x is 9 more than a multiple of 11, and
   - x is 2 more than a multiple of 13.

4. [**4**] Compute the smallest integer $n \geq 4$ such that $\binom{n}{4}$ ends in 4 or more zeroes (i.e. the rightmost four digits of $\binom{n}{4}$ are 0000).

5. [**5**] Let A be the greatest possible value of a product of positive integers that sums to 2014. Compute the sum of all bases and exponents in the prime factorization of A. For example, if $A = 7 \cdot 11^5$, the answer would be $7 + 11 + 5 = 23$.

6. [**6**] Let d be the greatest common divisor of $2^{30^{10}} - 2$ and $2^{30^{45}} - 2$. Find the remainder when d is divided by 2013.

7. [**7**] Define a "digitized number" as a ten-digit number $a_0a_1 \ldots a_9$ such that for $k = 0, 1, \ldots, 9$, a_k is equal to the number of times the digit k occurs in the number. Find the sum of all digitized numbers.

8. [**8**] What is the largest positive integer that cannot be expressed as a sum of non-negative integer multiples of $13, 17$ and 23?