1. Prove that \[
\frac{1}{a^2 + 2} + \frac{1}{b^2 + 2} + \frac{1}{c^2 + 2} \leq \frac{1}{6ab + c^2} + \frac{1}{6bc + a^2} + \frac{1}{6ca + b^2}
\]
for any positive real numbers \(a, b\) and \(c\) satisfying \(a^2 + b^2 + c^2 = 1\).

2. Let \(\gamma\) be the incircle of \(\triangle ABC\) (i.e. the circle inscribed in \(\triangle ABC\)) and \(I\) be the center of \(\gamma\). Let \(D, E\) and \(F\) be the feet of the perpendiculars from \(I\) to \(BC, CA\) and \(AB\) respectively. Let \(D'\) be the point on \(\gamma\) such that \(DD'\) is a diameter of \(\gamma\). Suppose the tangent to \(\gamma\) through \(D\) intersects the line \(EF\) at \(P\). Suppose the tangent to \(\gamma\) through \(D'\) intersects the line \(EF\) at \(Q\). Prove that \(\angle PIQ + \angle DAD' = 180^\circ\).

3. A graph consists of a set of vertices, some of which are connected by (undirected) edges. A star of a graph is a set of edges with a common endpoint. A matching of a graph is a set of edges such that no two have a common endpoint. Show that if the number of edges of a graph \(G\) is larger than \(2(k - 1)^2\), then \(G\) contains a matching of size \(k\) or a star of size \(k\).