PUMaC 2013

Individual Finals B

1. Let $a_1 = 2013$ and $a_{n+1} = 2013^{a_n}$ for all positive integers n. Let $b_1 = 1$ and $b_{n+1} = 2013^{2012 b_n}$ for all positive integers n. Prove that $a_n > b_n$ for all positive integers n.

2. Find all pairs of positive integers (a, b) such that

$$\frac{a^3 + 4b}{a + 2b^2 + 2a^2 b}$$

is a positive integer.

3. Find the smallest positive integer n with the following property: for every sequence of positive integers $a_1, a_2, ..., a_n$ with $a_1 + a_2 + ... + a_n = 2013$, there exist some (possibly one) consecutive term(s) in the sequence that add up to 70.