1. Let \( p \) be a prime number greater than 5. Prove that there exists a positive integer \( n \) such that \( p \) divides \( 20^n + 15^n - 12^n \).

2. Let \( a, b, c \) be real numbers such that \( a + b + c = abc \). Prove that \( \frac{1}{a^2+1} + \frac{1}{b^2+1} + \frac{1}{c^2+1} \geq \frac{3}{4} \).

3. Let \( ABC \) be a triangle with incenter \( I \), and let \( D \) be the foot of the angle bisector from \( A \) to \( BC \). Let \( \Gamma \) be the circumcircle of triangle \( BIC \), and let \( PQ \) be a chord of \( \Gamma \) passing through \( D \). Prove that \( AD \) bisects \( \angle PAQ \).

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers