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## Individual Finals A

1. Let  $p$  be a prime number greater than 5. Prove that there exists a positive integer  $n$  such that  $p$  divides  $20^n + 15^n - 12^n$ .
2. Let  $a, b, c$  be real numbers such that  $a + b + c = abc$ . Prove that  $\frac{1}{a^2+1} + \frac{1}{b^2+1} + \frac{1}{c^2+1} \geq \frac{3}{4}$ .
3. Let  $ABC$  be a triangle with incenter  $I$ , and let  $D$  be the foot of the angle bisector from  $A$  to  $BC$ . Let  $\Gamma$  be the circumcircle of triangle  $BIC$ , and let  $PQ$  be a chord of  $\Gamma$  passing through  $D$ . Prove that  $AD$  bisects  $\angle PAQ$ .

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers