1. Let $q$ be a fixed odd prime. A prime $p$ is said to be \textit{orange} if for every integer $a$ there exists an integer $r$ such that $r^q \equiv a \pmod{p}$. Prove that there are infinitely many orange primes.

2. Let $O_1, O_2, \ldots, O_{2012}$ be 2012 circles in the plane such that no circle intersects or contains any other circle and no two circles have the same radius. For each $1 \leq i < j \leq 2012$, let $P_{i,j}$ denote the point of intersection of the two external tangent lines to $O_i$ and $O_j$, and let $T$ be the set of all $P_{i,j}$ (so $|T| = \binom{2012}{2} = 2023066$). Suppose there exists a subset $S \subset T$ with $|S| = 2021056$ such that all points in $S$ lie on the same line. Prove that all points in $T$ lie on the same line.

3. Find, with proof, all pairs $(x, y)$ of integers satisfying the equation $3x^2 + 4 = 2y^3$.

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers