PUMaC 2011

Individual Finals B

1. If the plane is partitioned into a grid of congruent equilateral triangles, prove that there does not exist a square with vertices at the vertices of this grid.

2. Prove for irrational number α and positive integer n that

$$\left(\alpha + \sqrt{\alpha^2 - 1}\right)^{1/n} + \left(\alpha - \sqrt{\alpha^2 - 1}\right)^{1/n}$$

   is irrational.

3. In a k-player tournament for $k > 1$, every player plays every other player exactly once. Find with proof the smallest value of k such that it is possible that for any two players, there was a third player who beat both of them.

Please write complete, concise and clear proofs. Have fun! – PUMaC Problem Writers