PUMaC 2014

Combinatorics A

1. [**3**] What is the largest n such that a square cannot be partitioned into n smaller, non-overlapping squares?

2. [**3**] Assume you have a magical pizza in the shape of an infinite plane. You have a magical pizza cutter that can cut in the shape of an infinite line, but it can only be used 14 times. To share with as many of your friends as possible, you cut the pizza in a way that maximizes the number of finite pieces (the infinite pieces have infinite mass, so you can't lift them up). How many finite pieces of pizza do you have?

3. [**4**] You have three colors $\{red,\ blue,\ green\}$ with which you can color the faces of a regular octahedron (8 triangle sided polyhedron, which is two square based pyramids stuck together at their base), but you must do so in a way that avoids coloring adjacent pieces with the same color. How many different coloring schemes are possible? (Two coloring schemes are considered equivalent if one can be rotated to fit the other.)

4. [**4**] Amy has a 2×10 puzzle grid which she can use 1×1 and 1×2 (1 vertical, 2 horizontal) tiles to cover. How many ways can she exactly cover the grid without any tiles overlapping and without rotating the tiles?

5. [**5**] What is the size of the largest subset S' of $S = \{2^x 3^y 5^z : 0 \leq x, y, z \leq 4\}$ such that there are no distinct elements $p, q \in S'$ with $p|q$.

6. [**6**] Let $f(n)$ be the number of points of intersections of diagonals of a n-dimensional hypercube that is not the vertice of the cube. For example, $f(3) = 7$ because the intersection points of a cube's diagonals are at the centers of each face and the center of the cube. Find $f(5)$

7. [**7**] Ding and Jianing are playing a game. In this game, they use pieces of paper with 2014 positions, in which some permutation of the numbers $1, 2, ...2014$ are to be written. (Each number will be written exactly once). Ding fills in a piece of paper first. How many pieces of paper must Jianing fill in to ensure that at least one of her pieces of paper will have a permutation that has the same number as Ding's in at least one position?

8. [**8**] There are 60 friends who want to visit each others home during summer vacation. Everyday, they decide to either stay home or vist the home of everyone who stayed home that day. Find the minimum number of days required for everyone to have visted their friends' homes.