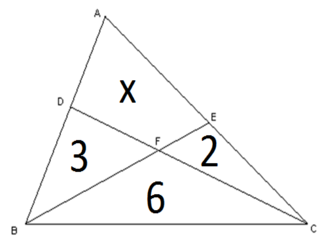
PUMaC 2014

Geometry A

1. [**3**] Let $x = \frac{p}{q}$ for p, q coprime. Find $p + q$



2. [**3**] Triangle ABC has lengths $AB = 20, AC = 14, BC = 22$. The median from B intersects AC at M and the angle bisector from C intersects AB at N and the median from B at P. Let $\frac{p}{q} = \frac{[AMPN]}{[ABC]}$ for positive integers p, q coprime. Note that $[ABC]$ denotes the area of triangle ABC. Find $p + q$

3. [**4**] Let O be the circumcenter of triangle ABC with circumradius 15. Let G be the centroid of ABC and let M be the midpoint of BC. If $BC = 18$ and $\angle MOA = 150^o$, find the area of OMG.

4. [**4**] Consider the cyclic quadrilateral with sides 1, 4, 8, 7 in that order. What is its circumdiameter? Let the answer be of the form $a\sqrt{b} + c$, for b square free. Find $a + b + c$

5. [**5**] There is a point D on side AC of acute triangle $\triangle ABC$. Let AM be the median drawn from A (so M is on BC) and CH be the altitude drawn from C (so H is on AB). Let I be the intersection of AM and CH, and let K be the intersection of AM and line segment BD. We know that $AK = 8$, $BK = 8$, and $MK = 6$. Find the length of AI.

6. [**6**] $\triangle ABC$ has side lengths $AB = 15$, $BC = 34$, and $CA = 35$. Let the circumcenter of ABC be O. Let D be the foot of perpendicular from C to AB. Let R be the foot of perpendicular from D to AC, and let W be the perpendicular foot from D to BC. Find the area of quadrilateral $CROW$.

7. [**7**] Let O be the center of a circle of radius 26, and let A, B be two distinct point on the circle, with M being the midpoint of AB. Consider point C for which $CO = 34$ and $\angle COM = 15^o$. Let N be the midpoint of CO. Suppose that $\angle ACB = 90^o$. Find MN.

8. [**8**] $ABCD$ is a cyclic quadrilateral with circumcenter O and circumradius 7. AB intersects CD at E, DA intersects CB at F. $OE = 13$, $OF = 14$. Let $\cos\angle FOE = \frac{p}{q}$, with p, q coprime. Find $p + q$.