Number Theory A

1. [3] Let \( f(x) = x^3 + ax^2 + bx + c \) have solutions that are distinct negative integers. If \( a + b + c = 2014 \), find \( c \).

2. [3] What is the last digit of \( 17^{17^{17}} \)?


4. [4] Find the sum of all positive integer \( x \) such that \( 3 \times 2^x = n^2 - 1 \) for some positive integer \( n \).

5. [5] Find the number of pairs of integer solution \( (x, y) \) that satisfies the equation

\[
(x - y + 2)(x - y - 2) = -(x - 2)(y - 2)
\]

6. [6] Given \( S = \{2, 5, 8, 11, 14, 17, 20\ldots\} \). Given that one can choose \( n \) different numbers from \( S \), \( \{A_1, A_2, \ldots A_n\} \), s.t. \( \sum_{i=1}^{n} \frac{1}{A_i} = 1 \). Find the minimum possible value of \( n \).

7. [7] Find the number of positive integers \( n \leq 2014 \) such that there exists integer \( x \) that satisfies the condition that \( \frac{x + n}{x - n} \) is an odd perfect square.

8. [8] Find all number sets \((a, b, c, d)\) s.t. \( 1 < a \leq b \leq c \leq d \), \( a, b, c, d \in \mathbb{N} \), and \( a^2 + b + c + d \), \( a + b^2 + c + d \), \( a + b + c^2 + d \) and \( a + b + c + d^2 \) are all square numbers. Sum the value of \( d \) across all solution set(s).