PUMαC 2014

Combinatorics B

1. **[3]** A girl and a guy are going to arrive at a train station. If they arrive within 10 minutes of each other, they will instantly fall in love and live happily ever after. But after 10 minutes, whichever one arrives first will fall asleep and they will be forever alone. The girl will arrive between 8 AM and 9 AM with equal probability. The guy will arrive between 7 AM and 8:30 AM, also with equal probability. Let $\frac{p}{q}$ for p, q coprime be the probability that they fall in love. Find $p + q$.

2. **[3]** A 100×100 grid is given as shown. We choose a certain number of cells such that exactly two cells in each row and column are selected. Find the sum of numbers in these cells.

1	2	...	99	100
101	102	...	199	200
...	...	...	...	...
9801	...	...	...	9900
9901	9902	...	9999	10000

3. **[4]** What is the largest n such that a square cannot be partitioned into n smaller, non-overlapping squares?

4. **[4]** Let there be 320 points arranged on a circle, labled $1, 2, 3, ..., 8, 1, 2, 3, ..., 8, ...$ in order. Line segments may only be drawn to connect points labelled with the same number. What thethe largest number of non-intersecting line segments one can draw? (Two segments sharing the same endpoint are considered to be intersecting).

5. **[5]** Amy has a 2×10 puzzle grid which she can use 1×1 and 1×2 (1 vertical, 2 horizontal) tiles to cover. How many ways can she exactly cover the grid without any tiles overlapping and without rotating the tiles?

6. **[6]** Consider an orange and black coloring of a 20×14 square grid. Let n be the number of coloring such that every row and column has an even number of orange square. Evaluate $\log_2 n$.

7. **[7]** Let $S = \{1, 2, 3, ..., 2014\}$. What is the largest subset of S that contains no two elements with a difference of 4 and 7?

8. **[8]** There are 60 friends who want to visit each other's home during summer vacation. Everyday, they decide to either stay home or vist the home of everyone who stayed home that day. Find the minimum number of days required for everyone to have visted their friends' homes.