PUMaC 2014

Geometry B

1. [**3**] Triangle ABC has lengths $AB = 20, AC = 14, BC = 22$. The median from B intersects AC at M and the angle bisector from C intersects AB at N and the median from B at P. Let $\frac{p}{q} = \frac{[AMPN]}{[ABC]}$ for positive integers p, q coprime. Note that $[ABC]$ denotes the area of triangle ABC. Find $p + q$

2. [**3**] Consider the pyramid $OABC$. Let the equilateral triangle ABC with side length 6 be the base. Also $9 = OA = OB = OC$. Let M be the midpoint of AB. Find the square of the distance from M to OC.

3. [**4**] As given in figure (not drawn to proportion), in $\triangle ABC$, $E \in AC$, $D \in AB$, $P = BE \cap CD$ Given that $S\triangle BPC = 12$, while the areas of $\triangle BPD$, $\triangle CPE$ and quadrilateral $AEPD$ are all the same, which is x. Find the value of x.

4. [**4**] Let O be the circumcenter of triangle ABC with circumradius 15. Let G be the centroid of ABC and let M be the midpoint of BC. If $BC = 18$ and $\angle MOA = 150^o$, find the area of OMG.

5. [**5**] Consider the cyclic quadrilateral with sides 1, 4, 8, 7 in that order. What is its circumdiameter? Let the answer be of the form $a\sqrt{b} + c$, for b square free. Find $a + b + c$

6. [**6**] There is a point D on side AC of acute triangle $\triangle ABC$. Let AM be the median drawn from A (so M is on BC) and CH be the altitude drawn from C (so H is on AB). Let I be the intersection of AM and CH, and let K be the intersection of AM and line segment BD. We know that $AK = 8$, $BK = 8$, and $MK = 6$. Find the length of AI.

7. [**7**] Consider quadrilateral $ABCD$. Given that $\angle DAC = 70$, $\angle BAC = 40$, $\angle BDC = 20$, $\angle CBD = 35$. Let P be the intersection of AC and BD. Find $\angle BPC$.

8. [**8**] $ABCD$ is a cyclic quadrilateral with circumcenter O and circumradius 7. AB intersects CD at E, DA intersects CB at F. $OE = 13$, $OF = 14$. Let $\cos\angle FOE = \frac{p}{q}$, with p, q coprime. Find $p + q$.