PUMaC 2014

Number Theory B

1. [**3**] Let $f(x) = x^3 + ax^2 + bx + c$ have solutions that are distinct negative integers. If $a + b + c = 2014$, find c.

2. [**3**] What is the last digit of $17^{17^{17^{17}}}$?

3. [**4**] Find the 3-digit positive integer that has the most divisors.

4. [**4**] Find the number of fractions in the following list that is in its lowest form. (ie. for $\frac{p}{q}$, $\gcd(p, q) = 1$.
$$\frac{1}{2014}, \frac{2}{2013}, \dots, \frac{1007}{1008}$$

5. [**5**] Find the sum of all positive integer x such that $3 \times 2^x = n^2 - 1$ for some positive integer n.

6. [**6**] Given $S = \{2, 5, 8, 11, 14, 17, 20 \dots\}$. Given that one can choose n different numbers from S, $\{A_1, A_2, \dots A_n\}$, s.t. $\sum_{i=1}^{n} \dfrac{1}{A_i} = 1$. Find the minimum possible value of n.

7. [**7**] How many permutations $p(n)$ of $\{1, 2, 3 \dots 35\}$ satisfy $a|b$ implies $p(a)|p(b)$?

8. [**8**] Find the number of positive integers $n \leq 2014$ such that there exists integer x that satisfies the condition that $\dfrac{x+n}{x-n}$ is an odd perfect square.