1. How many integer pairs \((a, b)\) with \(1 < a, b \leq 2015\) are there such that \(\log_a b\) is an integer?

2. There are real numbers \(a, b, c, d\) such that for all \((x, y)\) satisfying \(6y^2 = 2x^3 + 3x^2 + x\), if \(x_1 = ax + b\) and \(y_1 = cy + d\), then \(y_1^2 = x_1^3 - 36x_1\). What is \(a + b + c + d\)?

3. Find the sum of the non-repeated roots of the polynomial \(P(x) = x^6 - 5x^5 - 4x^4 - 5x^3 + 8x^2 + 7x + 7\).

4. Define the sequence \(a_i\) as follows: \(a_1 = 1, a_2 = 2015,\) and \(a_n = \frac{na_{n-1}}{a_{n-1} + na_{n-2}}\) for \(n > 2\). What is the least \(k\) such that \(a_k < a_{k-1}\)?

5. Since counting the numbers from 1 to 100 wasn’t enough to stymie Gauss, his teacher devised another clever problem that he was sure would stump Gauss. Defining \(\zeta_{15} = e^{2\pi i/15}\) where \(i = \sqrt{-1}\), the teacher wrote the 15 complex numbers \(\zeta_k\) for integer \(0 \leq k < 15\) on the board. Then, he told Gauss:

On every turn, erase two random numbers \(a, b\), chosen uniformly randomly, from the board and then write the term \(2ab - a - b + 1\) on the board instead. Repeat this until you have one number left. What is the expected value of the last number remaining on the board?

6. We define the function \(f(x, y) = x^3 + (y - 4)x^2 + (y^2 - 4y + 4)x + (y^3 - 4y^2 + 4y)\). Then choose any distinct \(a, b, c \in \mathbb{R}\) such that the following holds: \(f(a, b) = f(b, c) = f(c, a)\). Over all such choices of \(a, b, c\), what is the maximum value achieved by:

\[
\min(a^4 - 4a^3 + 4a^2, b^4 - 4b^3 + 4b^2, c^4 - 4c^3 + 4c^2)\]

7. We define the ridiculous numbers recursively as follows:

(a) 1 is a ridiculous number.

(b) If \(a\) is a ridiculous number, then \(\sqrt{a}\) and \(1 + \sqrt{a}\) are also ridiculous numbers.

A closed interval \(I\) is boring if

- \(I\) contains no ridiculous numbers, and
- There exists an interval \([b, c]\) containing \(I\) for which \(b\) and \(c\) are both ridiculous numbers.

The smallest non-negative \(l\) such that there does not exist a boring interval with length \(l\) can be represented in the form \(\frac{a + b\sqrt{c}}{d}\) where \(a, b, c, d\) are integers, \(\gcd(a, b, d) = 1\), and no integer square greater than 1 divides \(c\). What is \(a + b + c + d\)?

8. Let \(P(x)\) be a polynomial with positive integer coefficients and degree 2015. Given that there exists some \(\omega \in \mathbb{C}\) satisfying:

\[
\omega^{73} = 1 \quad \text{and} \quad P(\omega^{2015}) + P(\omega^{2015^2}) + P(\omega^{2015^3}) + \ldots + P(\omega^{2015^{72}}) = 0,
\]

what is the minimum possible value of \(P(1)\)?