1. For her daughter’s 12th birthday, Ingrid decides to bake a dodecagon pie in celebration. Unfortunately, the store does not sell dodecagon shaped pie pans, so Ingrid bakes a circular pie first and then trims off the sides in a way such that she gets the largest regular dodecagon possible. If the original pie was 8 inches in diameter, the area of pie that she has to trim off can be represented in square inches as \( a\pi - b \) where \( a, b \) are integers. What is \( a + b \)?

2. Terry the Tiger lives on a cube-shaped world with edge length 2. Thus he walks on the outer surface. He is tied, with a leash of length 2, to a post located at the center of one of the faces of the cube. The surface area of the region that Terry can roam on the cube can be represented as \( p\pi + \frac{a\sqrt{b} + c}{3} \) where \( a, b, c, p \) are integers, and no integer square greater than 1 divides \( b \) and \( p \) and \( c \) are coprime, and \( q > 0 \). What is \( p + q + a + b + c \)? (Terry can be at a location if the shortest distance along the surface of the cube between that point and the post is less than or equal to 2.)

3. Cyclic quadrilateral \( ABCD \) satisfies \( \angle ADC = 2 \cdot \angle BAD = 80^\circ \) and \( BC = CD \). Let the angle bisector of \( \angle BCD \) meet \( AD \) at \( P \). What is the measure, in degrees, of \( \angle BPD \)?

4. Find the largest \( r \) such that 4 balls each of radius \( r \) can be packed into a regular tetrahedron with side length 1. In a packing, each ball lies outside every other ball, and every ball lies inside the boundaries of the tetrahedron. If \( r \) can be expressed in the form \( \frac{\sqrt{a} + b}{c} \) where \( a, b, c \) are integers such that \( \gcd(b, c) = 1 \), what is \( a + b + c \)?

5. Let \( P, A, B, C \) be points on circle \( O \) such that \( C \) does not lie on arc \( BAP, \overline{PA} = 21, \overline{PB} = 56, \overline{PC} = 35 \) and \( m\angle BPC = 60^\circ \). Now choose point \( D \) on the circle such that \( C \) does not lie on arc \( BDP \) and \( BD = 39 \). What is \( AD \)?

6. Triangle \( ABC \) is inscribed in a unit circle \( \omega \). Let \( H \) be its orthocenter and \( D \) be the foot of the perpendicular from \( A \) to \( BC \). Let \( \Delta XYZ \) be the triangle formed by drawing the tangents to \( \omega \) at \( A, B, C \). If \( AH = HD \) and the side lengths of \( \Delta XYZ \) form an arithmetic sequence, the area of \( \Delta ABC \) can be expressed in the form \( \frac{p}{q} \pi \) for positive coprime integers \( p, q \). What is \( p + q \)?

7. Triangle \( ABC \) has \( \overline{AB} = \overline{AC} = 20 \) and \( \overline{BC} = 15 \). Let \( D \) be the point in \( \Delta ABC \) such that \( \Delta ADB \sim \Delta BDC \). Let \( l \) be a line through \( A \) and let \( BD \) and \( CD \) intersect \( l \) at \( P \) and \( Q \), respectively. Let the circumcircles of \( \Delta BDQ \) and \( \Delta CDP \) intersect at \( X \). The area of the locus of \( X \) as \( l \) varies can be expressed in the form \( \frac{p}{q} \pi \) for positive coprime integers \( p \) and \( q \). What is \( p + q \)?

8. The incircle of acute triangle \( ABC \) touches \( BC, AC, \) and \( AB \) at points \( D, E, \) and \( F \), respectively. Let \( P \) be the second intersection of line \( AD \) and the incircle. The line through \( P \) tangent to the incircle intersects \( AB \) and \( AC \) at points \( M \) and \( N \), respectively. Given that \( \overline{AB} = 8, \overline{AC} = 10, \) and \( \overline{AN} = 4 \), let \( \overline{AM} = \frac{a}{b} \) where \( a \) and \( b \) are positive coprime integers. What is \( a + b \)?