1. Alice is placing bishops on a 2015-by-2015 chessboard such that no two can attack one another.
(Bishops attack each other if they are on a diagonal.) Her friend Bob notices that he is not able
to place down a larger number of bishops such that any two still cannot attack one another.
If there are \( \prod p_i^{a_i} \), with \( a_i > 0 \) and \( p_i > 0 \) prime for all \( i \), ways Alice could have placed her
bishops, find \( \sum p_i + a_i \).

2. For an odd prime number \( p \), let \( S \) denote the following sum taken modulo \( p \):
\[
S \equiv \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{(p-2) \cdot (p-1)} \equiv \sum_{i=1}^{\frac{p-1}{2}} \frac{1}{(2i-1) \cdot 2i} \pmod{p}
\]
Prove that \( p^2 \mid 2^p - 2 \) if and only if \( S \equiv 0 \pmod{p} \).

3. Let \( I \) be the incenter of a triangle \( ABC \) with \( AB = 20 \), \( BC = 15 \), and \( BI = 12 \). Let \( CI \)
intersect the circumcircle \( \omega_1 \) of \( ABC \) at \( D \neq A \). Alice draws a line \( l \) through \( D \) that intersects
\( \omega_1 \) on the minor arc \( AC \) at \( X \) and the circumcircle \( \omega_2 \) of \( AIC \) at \( Y \) outside \( \omega_1 \). She notices
that she can construct a right triangle with side lengths \( ID, DX, \) and \( XY \). What is the length
of \( IY \)?