



## Combinatorics B

1. [3] A word is an ordered, non-empty sequence of letters, such as *word* or *wrod*. How many distinct 3-letter words can be made from a subset of the letters  $c, o, m, b, o$ , where each letter in the list is used no more than the number of times it appears?
2. [3] Jonathan has a magical coin machine which takes coins in amounts of 7, 8, and 9. If he puts in 7 coins, he gets 3 coins back; if he puts in 8, he gets 11 back; and if he puts in 9, he gets 4 back. The coin machine does not allow two entries of the same amount to happen consecutively. Starting with 15 coins, what is the minimum number of entries he can make to end up with 4 coins?
3. [4] Princeton's Math Club recently bought a stock for \$2 and sold it for \$9 thirteen days later. Given that the stock either increases or decreases by \$1 every day and never reached \$0, in how many possible ways could the stock have changed during those thirteen days?
4. [4] Andrew has 10 balls in a bag, each a different color. He randomly picks a ball from the bag 4 times, with replacement. The expected number of distinct colors among the balls he picks is  $\frac{p}{q}$ , where  $\gcd(p, q) = 1$  and  $p, q > 0$ . What is  $p + q$ ?
5. [5] Consider a random permutation of the set  $\{1, 2, \dots, 2015\}$ . In other words, for each  $1 \leq i \leq 2015$ ,  $i$  is sent to the element  $a_i$  where  $a_i \in \{1, 2, \dots, 2015\}$  and if  $i \neq j$ , then  $a_i \neq a_j$ . What is the expected number of ordered pairs  $(a_i, a_j)$  with  $i - j > 155$  and  $a_i - a_j > 266$ ?
6. [6] A number is *interesting* if it is a 6-digit integer that contains no zeros, its first 3 digits are strictly increasing, and its last 3 digits are non-increasing. What is the average of all interesting numbers?
7. [7] Alice has an orange 3-by-3-by-3 cube, which is comprised of 27 distinguishable, 1-by-1-by-1 cubes. Each small cube was initially orange, but Alice painted 10 of the small cubes completely black. In how many ways could she have chosen 10 of these smaller cubes to paint black such that every one of the 27 3-by-1-by-1 sub-blocks of the 3-by-3-by-3 cube contains at least one small black cube?
8. [8] In how many ways can 9 cells of a 6-by-6 grid be painted black such that no two black cells share a corner or an edge with each other?