# PUMaC 2015

## Individual Finals B

1. Alice places down $n$ bishops on a $2015 \times 2015$ chessboard such that no two bishops are attacking each other. (Bishops attack each other if they are on a diagonal.) Her friend Bob notices that he is not able to place down a larger number of bishops such that any two still cannot attack one another. Find, with proof, $n$.

2. On a circle $\omega_1$, four points $A$, $B$, $C$, $D$ lie in that order. Prove that $CD^2 = AC \cdot BC + AD \cdot BD$ if and only if at least one of $C$ and $D$ is the midpoint of arc $AB$.

3. For an odd prime number $p$, let $S$ denote the following sum taken modulo $p$:

$$S \equiv \frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \ldots + \frac{1}{(p-2) \cdot (p-1)} \equiv \sum_{i=1}^{\frac{p-1}{2}} \frac{1}{(2i-1) \cdot 2i} \pmod{p}$$

Prove that $p^2 | 2^p - 2$ if and only if $S \equiv 0 \pmod{p}$.