P U M .˙. C

Geometry A

1. Let $\triangle ABC$ be an equilateral triangle with side length 1 and let Γ the circle tangent to AB and AC at B and C, respectively. Let P be on side AB and Q be on side AC so that $PQ \parallel BC$, and the circle through A, P, and Q is tangent to Γ. If the area of $\triangle APQ$ can be written in the form $\frac{\sqrt{a}}{b}$ for positive integers a and b, where a is not divisible by the square of any prime, find $a+b$.

2. Let $ABCD$ be a square with side length 8. Let M be the midpoint of BC and let ω be the circle passing through M, A, and D. Let O be the center of ω, X be the intersection point (besides A) of ω with AB, and Y be the intersection point of OX and AM. If the length of OY can be written in simplest form as $\frac{m}{n}$, compute $m+n$.

3. Let $\mathcal{C}$ be a right circular cone with apex A. Let P_1, P_2, P_3, P_4 and P_5 be points placed evenly along the circular base in that order, so that $P_1P_2P_3P_4P_5$ is a regular pentagon. Suppose that the shortest path from P_1 to P_3 along the curved surface of the cone passes through the midpoint of AP_2. Let h be the height of $\mathcal{C}$, and r be the radius of the circular base of $\mathcal{C}$. If $\left(\frac{h}{r}\right)^2$ can be written in simplest form as $\frac{a}{b}$, find $a+b$.

4. Let $\triangle ABC$ be a triangle with integer side lengths such that $BC = 2016$. Let G be the centroid of $\triangle ABC$ and I be the incenter of $\triangle ABC$. If the area of $\triangle BGC$ equals the area of $\triangle BIC$, find the largest possible length of AB.

5. Let D, E, and F respectively be the feet of the altitudes from A, B, and C of acute triangle $\triangle ABC$ such that $AF = 28$, $FB = 35$ and $BD = 45$. Let P be the point on segment BE such that $AP = 42$. Find the length of CP.

6. In isosceles triangle ABC with base BC, let M be the midpoint of BC. Let P be the intersection of the circumcircle of $\triangle ACM$ with the circle with center B passing through M, such that $P \neq M$. If $\angle BPC = 135^\circ$, then $\frac{CP}{AP}$ can be written as $a + \sqrt{b}$ for positive integers a and b, where b is not divisible by the square of any prime. Find $a+b$.

7. Let $ABCD$ be a cyclic quadrilateral with circumcircle ω and let AC and BD intersect at X. Let the line through A parallel to BD intersect line CD at E and ω at $Y \neq A$. If $AB = 10$, $AD = 24$, $XA = 17$, and $XB = 21$, then the area of $\triangle DEY$ can be written in simplest form as $\frac{m}{n}$. Find $m+n$.

8. Let $\triangle ABC$ have side lengths $AB = 4, BC = 6, CA = 5$. Let M be the midpoint of BC and let P be the point on the circumcircle of $\triangle ABC$ such that $\angle MPA = 90^\circ$. Let D be the foot of the altitude from B to AC, and let E be the foot of the altitude from C to AB. Let PD and PE intersect line BC at X and Y, respectively. Compute the square of the area of $\triangle AXY$.