1. Let $\triangle ABC$ be an equilateral triangle with side length 1 and let $\Gamma$ the circle tangent to $AB$ and $AC$ at $B$ and $C$, respectively. Let $P$ be on side $AB$ and $Q$ be on side $AC$ so that $PQ \parallel BC$, and the circle through $A$, $P$, and $Q$ is tangent to $\Gamma$. If the area of $\triangle APQ$ can be written in the form $\sqrt{\frac{a}{b}}$ for positive integers $a$ and $b$, where $a$ is not divisible by the square of any prime, find $a + b$.

2. Let $ABCD$ be a square with side length 8. Let $M$ be the midpoint of $BC$ and let $\omega$ be the circle passing through $M$, $A$, and $D$. Let $O$ be the center of $\omega$, $X$ be the intersection point (besides $A$) of $\omega$ with $AB$, and $Y$ be the intersection point of $OX$ and $AM$. If the length of $OY$ can be written in simplest form as $\frac{m}{n}$, compute $m + n$.

3. Let $C$ be a right circular cone with apex $A$. Let $P_1$, $P_2$, $P_3$, $P_4$, and $P_5$ be points placed evenly along the circular base in that order, so that $P_1P_2P_3P_4P_5$ is a regular pentagon. Suppose that the shortest path from $P_1$ to $P_3$ along the curved surface of the cone passes through the midpoint of $AP_2$. Let $h$ be the height of $C$, and $r$ be the radius of the circular base of $C$. If $(\frac{h}{r})^2$ can be written in simplest form as $\frac{a}{b}$, find $a + b$.

4. Let $\triangle ABC$ be a triangle with integer side lengths such that $BC = 2016$. Let $G$ be the centroid of $\triangle ABC$ and $I$ be the incenter of $\triangle ABC$. If the area of $\triangle BGC$ equals the area of $\triangle BIC$, find the largest possible length of $AB$.

5. Let $D$, $E$, and $F$ respectively be the feet of the altitudes from $A$, $B$, and $C$ of acute triangle $\triangle ABC$ such that $AF = 28$, $FB = 35$ and $BD = 45$. Let $P$ be the point on segment $BE$ such that $\triangle APB$ is equilateral. If the area of $\triangle ACD$ can be written in simplest form as $\frac{m}{n}$, find $m + n$.

6. In isosceles triangle $ABC$ with base $BC$, let $M$ be the midpoint of $BC$. Let $P$ be the intersection of the circumcircle of $\triangle ACM$ with the circle with center $B$ passing through $M$, such that $P \neq M$. If $\angle BPC = 135^\circ$, then $\frac{CP}{MP}$ can be written as $a + \sqrt{b}$ for positive integers $a$ and $b$, where $b$ is not divisible by the square of any prime. Find $a + b$.

7. Let $ABCD$ be a cyclic quadrilateral with circumcircle $\omega$ and let $AC$ and $BD$ intersect at $X$. Let the line through $A$ parallel to $BD$ intersect line $CD$ at $E$ and $\omega$ at $Y \neq A$. If $AB = 10$, $AD = 24$, $XA = 17$, and $XB = 21$, then the area of $\triangle DEY$ can be written in simplest form as $\frac{m}{n}$. Find $m + n$.

8. Let $\triangle ABC$ have side lengths $AB = 4$, $BC = 6$, $CA = 5$. Let $M$ be the midpoint of $BC$ and let $P$ be the point on the circumcircle of $\triangle ABC$ such that $\angle MPB = 90^\circ$. Let $D$ be the foot of the altitude from $B$ to $AC$, and let $E$ be the foot of the altitude from $C$ to $AB$. Let $PD$ and $PE$ intersect line $BC$ at $X$ and $Y$, respectively. Compute the square of the area of $\triangle AXY$. 