P U M ∴ C

Algebra B

1. If x is a positive real number such that $(x^2-1)^2-1=9800$, compute x.

2. Let $a_1=20$, $a_2=16$, and for $k\geq 3$, let $a_k=\sqrt[3]{k-a_{k-1}^3-a_{k-2}^3}$. Compute $a_1^3+a_2^3+\cdots+a_{10}^3$.

3. Bob draws the graph of $y=x^3-13x^2+40x+25$ and is dismayed to find out that it only has one root. Alice comes to the rescue, translating (without rotating or dilating) the axes so that the origin is at the point that used to be $(-20,16)$. This new graph has three x-intercepts; compute their sum.

4. Let $f(x)=15x-2016$. If $f(f(f(f(f(x)))))=f(x)$, find the sum of all possible values of x.

5. For positive real numbers x and y, let $f(x,y)=x^{\log_2 y}$. The sum of the solutions to the equation
$$4096f(f(x,x),x)=x^{13}$$
can be written in simplest form as $\frac{m}{n}$. Compute $m+n$.

6. Suppose that P is a polynomial with integer coefficients such that $P(1)=2$, $P(2)=3$ and $P(3)=2016$. If N is the smallest possible positive value of $P(2016)$, find the remainder when N is divided by 2016.

7. Define a sequence a_i as follows: $a_1=181$ and for $i\geq 2$, $a_i=a_{i-1}^2-1$ if a_{i-1} is odd and $a_i=a_{i-1}/2$ if a_{i-1} is even. Find the least i such that $a_i=0$.

8. Let S_P be the set of all polynomials P with complex coefficients, such that $P(x^2)=P(x)P(x-1)$ for all complex numbers x. Suppose P_0 is the polynomial in S_P of maximal degree such that $P_0(1)\mid 2016$. Find $P_0(10)$.