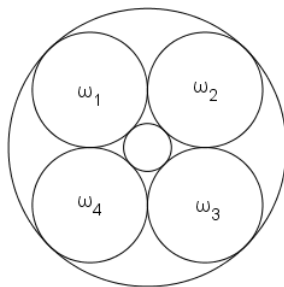




## Geometry B

1. A circle of radius 1 has four circles  $\omega_1, \omega_2, \omega_3,$  and  $\omega_4$  of equal radius internally tangent to it, so that  $\omega_1$  is tangent to  $\omega_2$ , which is tangent to  $\omega_3$ , which is tangent to  $\omega_4$ , which is tangent to  $\omega_1$ , as shown. The radius of the circle externally tangent to  $\omega_1, \omega_2, \omega_3,$  and  $\omega_4$  has radius  $r$ . If  $r = a - \sqrt{b}$  for positive integers  $a$  and  $b$ , compute  $a + b$ .



2. Let  $\triangle ABC$  be an equilateral triangle with side length 1 and let  $\Gamma$  the circle tangent to  $AB$  and  $AC$  at  $B$  and  $C$ , respectively. Let  $P$  be on side  $AB$  and  $Q$  be on side  $AC$  so that  $PQ \parallel BC$ , and the circle through  $A, P,$  and  $Q$  is tangent to  $\Gamma$ . If the area of  $\triangle APQ$  can be written in the form  $\frac{\sqrt{a}}{b}$  for positive integers  $a$  and  $b$ , where  $a$  is not divisible by the square of any prime, find  $a + b$ .
3. Let  $ABCD$  be a square with side length 8. Let  $M$  be the midpoint of  $BC$  and let  $\omega$  be the circle passing through  $M, A,$  and  $D$ . Let  $O$  be the center of  $\omega, X$  be the intersection point (besides  $A$ ) of  $\omega$  with  $AB$ , and  $Y$  be the intersection point of  $OX$  and  $AM$ . If the length of  $OY$  can be written in simplest form as  $\frac{m}{n}$ , compute  $m + n$ .
4. Let  $\mathcal{C}$  be a right circular cone with apex  $A$ . Let  $P_1, P_2, P_3, P_4$  and  $P_5$  be points placed evenly along the circular base in that order, so that  $P_1P_2P_3P_4P_5$  is a regular pentagon. Suppose that the shortest path from  $P_1$  to  $P_3$  along the curved surface of the cone passes through the midpoint of  $AP_2$ . Let  $h$  be the height of  $\mathcal{C}$ , and  $r$  be the radius of the circular base of  $\mathcal{C}$ . If  $\left(\frac{h}{r}\right)^2$  can be written in simplest form as  $\frac{m}{n}$ , find  $m + n$ .
5. Let  $V$  be the volume of the octahedron  $ABCDEF$  with  $A$  and  $F$  opposite,  $B$  and  $E$  opposite, and  $C$  and  $D$  opposite, such that  $AB = AE = EF = BF = 13, BC = DE = BD = CE = 14,$  and  $CF = CA = AD = FD = 15$ . If  $V = a\sqrt{b}$  for positive integers  $a$  and  $b$ , where  $b$  is not divisible by the square of any prime, find  $a + b$ .
6. Let  $D, E,$  and  $F$  respectively be the feet of the altitudes from  $A, B,$  and  $C$  of acute triangle  $\triangle ABC$  such that  $AF = 28, FB = 35$  and  $BD = 45$ . Let  $P$  be the point on segment  $BE$  such that  $AP = 42$ . Find the length of  $CP$ .
7. In isosceles triangle  $ABC$  with base  $BC$ , let  $M$  be the midpoint of  $BC$ . Let  $P$  be the intersection of the circumcircle of  $\triangle ACM$  with the circle with center  $B$  passing through  $M$ , such that  $P \neq M$ . If  $\angle BPC = 135^\circ$ , then  $\frac{CP}{AP}$  can be written as  $a + \sqrt{b}$  for positive integers  $a$  and  $b$ , where  $b$  is not divisible by the square of any prime. Find  $a + b$ .
8. Let  $ABCD$  be a cyclic quadrilateral with circumcircle  $\omega$  and let  $AC$  and  $BD$  intersect at  $X$ . Let the line through  $A$  parallel to  $BD$  intersect line  $CD$  at  $E$  and  $\omega$  at  $Y \neq A$ . If  $AB = 10, AD = 24, XA = 17,$  and  $XB = 21$ , then the area of  $\triangle DEY$  can be written in simplest form as  $\frac{m}{n}$ . Find  $m + n$ .